Question 9 **(50 marks)**

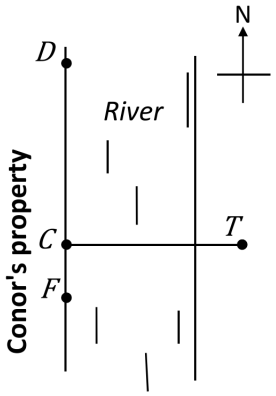

Figure 1

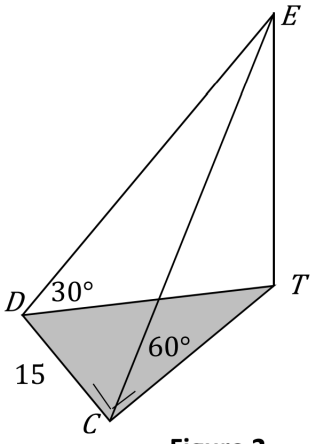

Figure 2

Conor's property is bounded by the straight bank of a river, as shown in **Figure 1** above.
T is the base of a vertical tree that is growing near the opposite bank of the river.
$|TE|$ is the height of the tree, as shown in **Figure 2** above.
From the point C, which is due west of the tree, the angle of elevation of E, the top of the tree, is $60°$.
From the point D, which is 15 m due north of C, the angle of elevation of E is $30°$ (see **Figure 2**).
The land on both sides of the river is flat and at the same level.

(a) Use triangle ECT, to express $|TE|$ in the form $\sqrt{a}|CT|$ metres, where $a \in \mathbb{N}$.

(b) Show that $|TE|$ may also be expressed as $\sqrt{\frac{225+|CT|^2}{3}}$ metres.

(c) Hence find $|CT|$, the distance from the base of the tree to the bank of the river at Conor's side. Give your answer correct to 1 decimal place.

(d) Find $|TE|$, the height of the tree. Give your answer correct to 1 decimal place.

(e) The tree falls across the river and hits the bank at Conor's side at the point F. Find the maximum size of the angle FTC. Give your answer in degrees, correct to 1 decimal place.

(f) If the tree was equally likely to fall in any direction, find the probability that it would hit the bank at Conor's side, when it falls.
Give your answer as a percentage, correct to 1 decimal place.

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$\tan 60° = \frac{\lvert TE\rvert}{\lvert CT\rvert}$ $\sqrt{3}\lvert CT\rvert = \lvert TE\rvert$	**Scale 10B (0, 5, 10)** *Partial Credit:* • $\tan 60°$ • effort to express $\lvert TE\rvert$ in terms of another side of the triangle
(b)	$\tan 30° = \frac{\lvert TE\rvert}{\lvert DT\rvert}$ $\lvert TE\rvert = \lvert DT\rvert \frac{1}{\sqrt{3}}$ $\lvert TE\rvert = \frac{\sqrt{225+\lvert CT^2\rvert}}{\sqrt{3}}$ $\lvert TE\rvert = \sqrt{\frac{225 + \lvert CT\rvert^2}{3}}$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • $\tan 30°$ • Use of Pythagoras for $\lvert DT\rvert$ • Effort at expressing $\lvert DT\rvert$ in terms of another side of ΔDET *High Partial Credit:* • $\lvert TE\rvert = \lvert DT\rvert \frac{1}{\sqrt{3}}$
(c)	$\sqrt{3}\lvert CT\rvert = \sqrt{\frac{225 + \lvert CT\rvert^2}{3}}$ $\lvert CT\rvert = \sqrt{\frac{225}{8}}$ $= 5{\cdot}3033 \text{ m}$ $= 5{\cdot}3 \text{ m}$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • equates both expressions *High Partial Credit:* • Isolate $\lvert CT\rvert$ in equation

Q9		Marking Notes
(d)	$\lvert TE\rvert = \sqrt{3}\lvert CT\rvert = 9{\cdot}17986\text{ m} = 9{\cdot}2\text{ m}$	**Scale 10B (0, 5, 10)** *Low Partial Credit* • Substitution into formula for $\lvert TE\rvert$
(e)	$\cos\theta = \frac{\lvert CT\rvert}{\lvert FT\rvert} = \frac{\lvert CT\rvert}{\lvert TE\rvert} = \frac{\lvert CT\rvert}{\sqrt{3}\lvert CT\rvert} = \frac{1}{\sqrt{3}}$ $\theta = 54{\cdot}7$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Some relevant substitution for $\cos\theta$ *High Partial Credit:* • Formula for $\cos\theta$ substituted in terms of $\lvert CT\rvert$
(f)	$P = \frac{(54{\cdot}7)(2)}{360}$ $= 0{\cdot}3038$ $= 30{\cdot}4$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • (Answer to **part (e)**)×2 • $360°$ *High Partial Credit:* • P fully formulated