Question 2 **(25 marks)**

(a) The first three terms of a geometric series are x^2, $5x - 8$, and $x + 8$, where $x \in \mathbb{R}$.
Use the common ratio to show that $x^3 - 17x^2 + 80x - 64 = 0$.

(b) If $f(x) = x^3 - 17x^2 + 80x - 64$, $x \in \mathbb{R}$, show that $f(1) = 0$, **and** find another value of x for which $f(x) = 0$.

(c) In the case of one of the values of x from part **(b)**, the terms in part **(a)** will generate a geometric series with a finite sum to infinity.
Find this value of x and **hence** find the sum to infinity.

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{5x-8}{x^2} = \frac{x+8}{5x-8}$ $(5x-8)^2 = x^2(x+8)$ $25x^2 - 80x + 64 = x^3 + 8x^2$ $x^3 - 17x^2 + 80x - 64 = 0$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $\frac{5x-8}{x^2}$ or $\frac{x+8}{5x-8}$ Some effort at finding r in a geometric sequence (must use at least one of the terms) $r = \frac{T_n}{T_{n-1}}$ or similar *High Partial Credit:* $\frac{5x-8}{x^2} = \frac{x+8}{5x-8}$ $(5x-8)^2$ and $x^2(x+8)$ *0 credit*: Treats as an arithmetic sequence
(b)	$f(x) = x^3 - 17x^2 + 80x - 64$ $f(1) = (1)^3 - 17(1)^2 + 80(1) - 64 = 0$ $\Rightarrow (x-1)$ is a factor $x^3 - 17x^2 + 80x - 64 = 0$ $x^2(x-1) - 16x(x-1) + 64(x-1)$ $x^2 - 16x + 64 = 0$ $(x-8)(x-8) = 0$ $x = 8$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Shows $f(1) = 0$ Any correct substitution *High Partial Credit:* Quotient in quadratic form found Accept $x = 8$ without work if $f(1) = 0$ has been shown

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<td>(c)</td>
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<u>$x = 1$</u>
$1^2, \quad 5(1) - 8, \; 1 + 8$
$1, -3, \; 9$ which doesn't have a sum to infinity $(|r| > 1)$

<u>$x = 8$</u>
$8^2, \quad 5(8) - 8, \quad 8 + 8$
$64, 32, 16 \ldots . \; a = 64$ and $r = \frac{1}{2}$
$S_\infty = \frac{a}{1-r} = \frac{64}{1-\frac{1}{2}} = \frac{64}{\frac{1}{2}} = 128$
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Scale 5C (0, 3, 4, 5)
Low Partial Credit:
Substitution used to identify $x = 8$ as the required value
Substitution used to exclude $x = 1$ as the required value
Finds $\frac{a}{1-r}$ for $x = 1$
$S_\infty = \dfrac{x^2}{1 - \dfrac{5x-8}{x^2}}$
Relevant substitution into correct formula

High Partial Credit:
GP identified (a and r)
If the candidate works with both $x = 1$ and $x = 8$ but fails to eliminate $x = 1$ or chooses the incorrect answer

Note: if $|r| > 1$ then Low Partial Credit at most
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