Question 3 **(25 marks)**

(a) Let $h(x) = \cos(2x)$, where $x \in \mathbb{R}$.
A tangent is drawn to the graph of $h(x)$ at the point where $x = \frac{\pi}{3}$.
Find the angle that this tangent makes with the positive sense of the x-axis.

(b) Find the average value of $h(x)$ over the interval $0 \leq x \leq \frac{\pi}{4}$, $x \in \mathbb{R}$.
Give your answer in terms of π.

<table>
<tr><th>Q3</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$h'(x) = -2\sin(2x)$
At $x = \frac{\pi}{3}$: $h'\left(\frac{\pi}{3}\right) = -2\sin\left(\frac{2\pi}{3}\right)$
$= -2\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$\tan\theta = -\sqrt{3}$

$\theta = 120°$</td><td>Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Differentiation indicated
Use of 2

Mid Partial Credit:
Derivative found

High Partial Credit:
$\tan\theta =$ evaluated derivative
$\theta = -60°$

Note: Must use differentiation to gain any credit
Note: If integration symbol appears then 0 credit</td></tr>
<tr><td>(b)</td><td>$\frac{1}{\frac{\pi}{4} - 0}\int_0^{\frac{\pi}{4}} \cos(2x)\,dx$
$= \frac{4}{\pi}\left(\frac{\sin(2x)}{2}\right)\Big|_0^{\frac{\pi}{4}}$
$= \frac{4}{\pi}\left(\frac{\sin\frac{\pi}{2}}{2} - \frac{\sin 0}{2}\right)$
$= \frac{4}{\pi}\left(\frac{1}{2}\right) = \frac{2}{\pi}$</td><td>Scale 15D (0, 5, 7, 11, 15)
Low Partial Credit:
Integration indicated

Mid Partial Credit:
$\cos 2x$ integrated correctly $\left(\frac{\sin(2x)}{2}\right)$
$-2\sin 2x$ and finishes correctly

High Partial Credit:
Substitutes limits into integral and stops
Integral evaluated at $x = \frac{\pi}{4}$ (i.e. omits $\frac{1}{\frac{\pi}{4}-0}$) and finishes

Note: errors in integration could include
An error in the trig function (including sign)
An error in the angle
An error in the application of the chain rule

Note: Must have integration to gain any credit</td></tr>
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