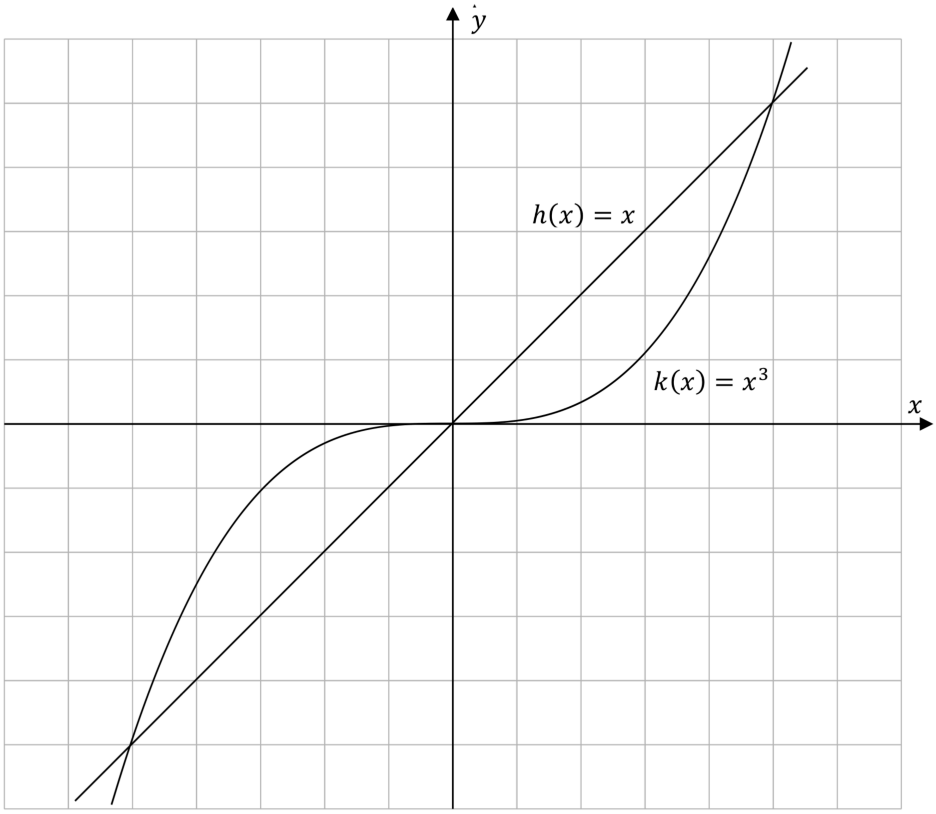
Question 6 **(25 marks)**

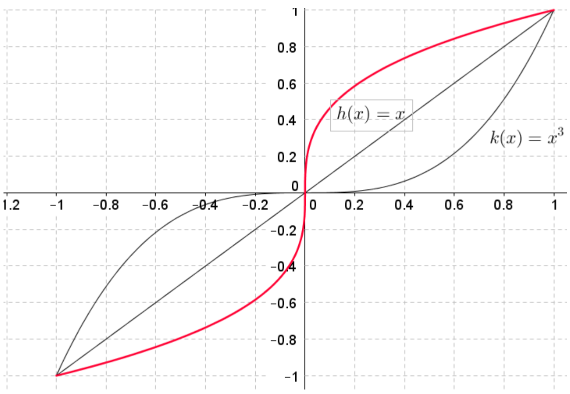
Parts of the graphs of the functions $h(x) = x$ and $k(x) = x^3$, $x \in \mathbb{R}$, are shown in the diagram below.



(a) Find the co-ordinates of the points of intersection of the graphs of the two functions.

(b) **(i)** Find the total area enclosed between the graphs of the two functions.

(ii) On the diagram on the previous page, using symmetry or otherwise, draw the graph of k^{-1}, the inverse function of k.

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$x^3 = x$ $\Longrightarrow x^3 - x = 0$ $\Longrightarrow x(x^2 - 1) = 0$ $x(x-1)(x+1) = 0$ $x = 0$ or $x = \pm 1$ (−1, −1), (0, 0), (1, 1)	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Equation written One correct solution from the graph Solution of the form (a, a) where $a \neq 0, 1$ *High Partial Credit:* Equation factorised (3 factors) 2 correct points x values only
(b) (i)	$2\int_0^1 x - x^3 dx$ $= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$ =2 $\left[\frac{1}{2} - \frac{1}{4} - 0\right] =$ $\frac{1}{2}$ unit^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Integral indicated One relevant area found *High Partial Credit:* Integral evaluated at $x = 1$ (upper limit) $\int_{-1}^{1} x - x^3 dx = 0$
(b) (ii)		**Scale 5B (0, 2, 5)** *Partial Credit:* Incomplete image 2 correct image points $k^{-1}(x) = x^{\frac{1}{3}}$