Question 7 **(55 marks)**

The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

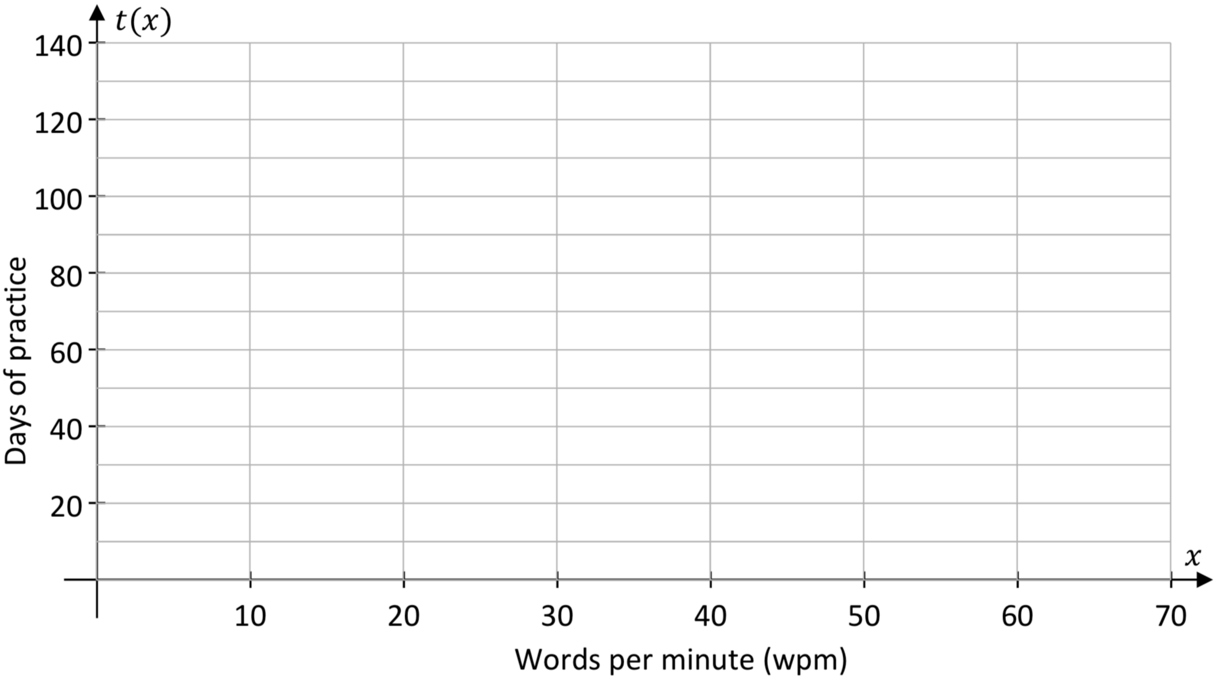
$$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right], \text{ where } 0 \leq x \leq 70, x \in \mathbb{R}, \text{ and } k \text{ is a constant.}$$

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of k and **hence** show that $k = -62{\cdot}5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice. Give your answer correct to the nearest whole number.

(c) Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								



(d) A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1{\cdot}5x$.
Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

(e) Let $h(x) = p(x) - t(x)$.

(i) Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

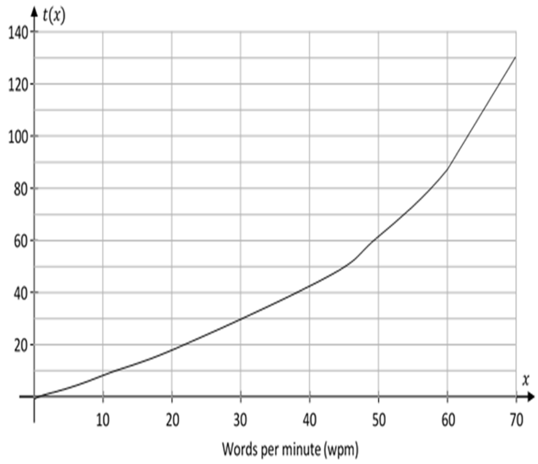
(ii) Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.
Give your answer correct to the nearest whole number.

Q7	Model Solution – 55 Marks	Marking Notes
(a)	$35{\cdot}96 = k\ln\left(1 - \frac{35}{80}\right)$ $35{\cdot}96 = k\ln\left(\frac{45}{80}\right)$ $k = \dfrac{35{\cdot}96}{\ln\left(\frac{45}{80}\right)}$ $k = -62{\cdot}5$ to one place of decimals	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Effort at transposing Some substitution into function Full substitution and stops *High Partial Credit:* Function written in terms of k and fully substituted One incorrect substitution worked correctly and with some reference to $k \neq -62{\cdot}5$
(b)	$100 = -62{\cdot}5 \ln\left(1 - \frac{x}{80}\right)$ $\dfrac{100}{-62{\cdot}5} = \ln\left(1 - \frac{x}{80}\right)$ $e^{\frac{100}{-62\cdot5}} = 1 - \frac{x}{80}$ $x = -80(e^{\frac{100}{-62\cdot5}} - 1)$ $x = 64$ wpm (To the nearest whole number)	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some substitution into function Trial and improvement (more than 1 iteration) Correct answer without work *High Partial Credit:* $e^{\frac{100}{-62\cdot5}} = 1 - \frac{x}{80}$ Equation rewritten in terms of x or $\frac{x}{80} =$

(c)

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)	0	8	18	29	43	61	87	130

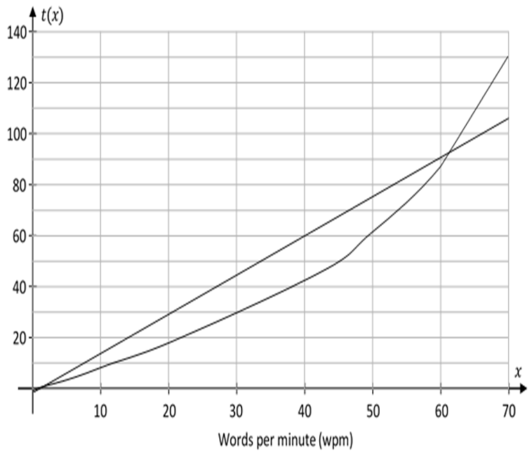
(c)



Scale 20D (0, 5, 10, 15, 20)
Low Partial Credit:
One entry correct
One plot (from candidates table) correct

Mid Partial Credit:
4 entries correct and 4 plots of table values

High Partial Credit:
All plots consistent with candidates table values (with at least 1 correct value)
Table correct but incorrect plots

(d)		**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* One point on line identified One point (not origin) plotted *High Partial Credit:* 2 points on line identified and plotted
(e) (i)	Approx 62 wpm	**Scale 5B(0, 2, 5)** *Partial Credit:* Point of intersection indicated on graph $h(x)$ written in terms of x **Tolerance**: $\pm$ 2 wpm
(e) (ii)	For Maximum Value: Set $h'(x) = 0$ $h(x) = 1{\cdot}5x + 62{\cdot}5 \ln\left(1 - \frac{x}{80}\right)$ $h'(x) = 1{\cdot}5 + 62{\cdot}5\left(\frac{1}{1-\frac{x}{80}}\right) \times \left(-\frac{1}{80}\right)$ $= 0$ $\frac{62{\cdot}5}{80-x} = 1{\cdot}5$ $x = 80 - \frac{62{\cdot}5}{1{\cdot}5}$ $x = 38{\cdot}3 = 38$ words $h\left(38\frac{1}{3}\right) = 1{\cdot}5\left(38\frac{1}{3}\right)$ $+ 62{\cdot}5 \ln\left(1 - \frac{38\frac{1}{3}}{80}\right) = 16{\cdot}73$ = 17 days	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any correct differentiation $h(x) = 1{\cdot}5x + 62{\cdot}5 \ln\left(1 - \frac{x}{80}\right)$ $h'(x) = 0$ *High Partial Credit:* Differentiation correct but un-simplified Value for x and stops