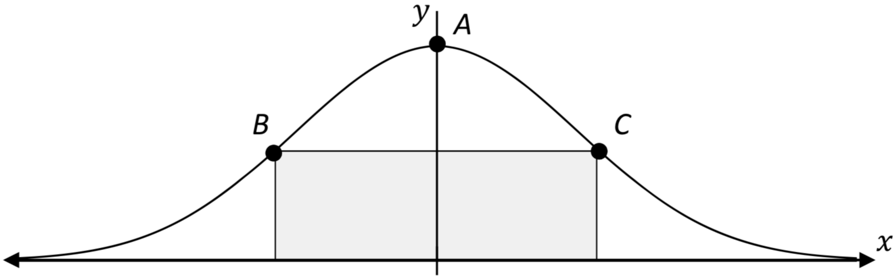
Question 8 **(40 marks)**

The graph of the **symmetric** function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ is shown below.



(a) Find the co-ordinates of *A*, the point where the graph intersects the y-axis. Give your answer in terms of π.

(b) The co-ordinates of *B* are $\left(-1, \frac{1}{\sqrt{2\pi e}}\right)$. Find the area of the shaded rectangle in the diagram above. Give your answer correct to 3 decimal places.

(c) Use calculus to show that $f(x)$ is decreasing at *C*.

(d) Show that the graph of $f(x)$ has a point of inflection at *B*.

Q8	Model Solution – 40 Marks	Marking Notes
(a)	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ At $x = 0$: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0)^2}$ $= \frac{1}{\sqrt{2\pi}}(1)$ $\therefore\ (0, \frac{1}{\sqrt{2\pi}})$ is the y intercept	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $x = 0$ Value for x substituted into $f(x)$ *High Partial Credit:* $\frac{1}{\sqrt{2\pi}}$ *Full credit* – 1: (0, 0·3989)
(b)	$\text{Area} = \left[(2)\left(\frac{1}{\sqrt{2\pi e}}\right)\right] = 0{\cdot}4839$ $= 0{\cdot}484$ Units^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* length $= 2$ Width $=$ [y co-ordinate] *High Partial Credit:* $\left[(1)(\frac{1}{\sqrt{2\pi e}})\right]$ *Full credit* –1: Area = $-0{\cdot}484$ *Zero Credit*: Integrating original function
(c)	$C(1, \frac{1}{\sqrt{2\pi e}})$ due to symmetry $f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $At\ x = 1:\ f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(1)^2}(-1) < 0$ $\left[= -\frac{1}{\sqrt{2\pi e}} \quad (-0{\cdot}\ 24197) \ < 0 \ \right]$ $\Longrightarrow$ Decreasing	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $x = 1$ identified Some correct differentiation Indicates significance of $\frac{dy}{dx} < 0$ *High Partial Credit:* Derivative found

(d)	$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $f''(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-1)$ $\qquad + (-x)\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(-x)$ $= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^2 - 1)$ $f''(-1) = 0 \text{ as } 1^2 - 1 = 0$ $\Longrightarrow \text{point of inflection at } x = -1$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $f'(x)$ transferred or found Mention of $f''(x)$ Identifies $x = -1$ *Mid Partial Credit:* $f''(x)$ identified and some correct differentiation *High Partial Credit:* $f''(x)$ found **Note:** if the product rule **and** chain rule are not applied in finding $f''(x)$ then the candidate can be awarded mid partial credit at most