Question 7 **(55 marks)**

The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

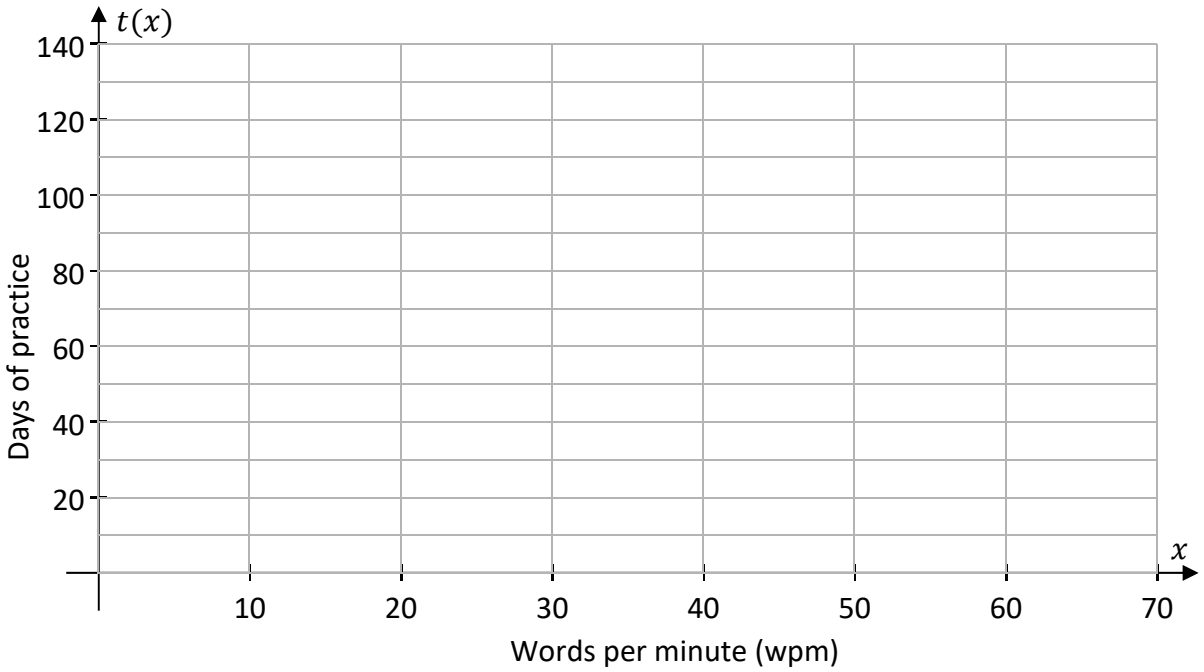
$$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right], \text{ where } 0 \leq x \leq 70, x \in \mathbb{R}, \text{ and } k \text{ is a constant.}$$

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of k and **hence** show that $k = -62{\cdot}5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice.
Give your answer correct to the nearest whole number.

(c) Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								



(d) A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1{\cdot}5x$.
Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

(e) Let $h(x) = p(x) - t(x)$.

(i) Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

(ii) Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.
Give your answer correct to the nearest whole number.