## Question 1

(25 marks)
In a competition Mary has a probability of $\frac{1}{20}$ of winning, a probability of $\frac{1}{10}$ of finishing in second place, and a probability of $\frac{1}{4}$ of finishing in third place. If she wins the competition she gets $€ 9000$. If she comes second she gets $€ 7000$ and if she comes third she gets $€ 3000$. In all other cases she gets nothing. Each participant in the competition must pay $€ 2000$ to enter.
(a) Find the expected value of Mary's loss if she enters the competition.
(b) Each of the 3 prizes in the competition above is increased by the same amount ( $£ x$ ) but the entry fee is unchanged.
For example, if Mary wins the competition now, she would get $€(9000+x)$.
Mary now expects to break even.
Find the value of $x$.

| Q1 | Model Solution - 25 Marks | Marking Notes |
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| (a) | $\begin{gathered} \frac{1}{20}(9000)+\frac{1}{10}(7000)+\frac{1}{4}(3000) \\ =1900 \\ E(x)=2000-1900=100 \end{gathered}$ <br> Or $\begin{aligned} & E(x)=\frac{1}{20}(-7000)+\frac{1}{10}(-5000) \\ & \quad+\frac{1}{4}(-1000)+\frac{3}{5}(2000) \\ & =-350-500-250+1200=100 \end{aligned}$ <br> So expected gain for organisers of competition and therefore a loss for Mary of 100 | Scale 15C (0, 4, 11, 15) <br> Low Partial Credit: <br> $E(x)$ partially formulated (1 or 2 terms) <br> High Partial Credit: <br> $E(x)$ fully formulated (sum of all three/all four terms) |


| (b) | $\begin{gathered} \frac{1}{20}(9000+x)+\frac{1}{10}(7000+x) \\ +\frac{1}{4}(3000+x)=2000 \\ \left(1900+\frac{8}{20} x\right)=2000 \\ \frac{8}{20} x=100 \\ x=250 \end{gathered}$ <br> Or <br> From (a) to break even it will take $€ 100$. $\begin{gathered} \frac{x}{20}+\frac{x}{10}+\frac{x}{4}=100 \\ \frac{x+2 x+5 x}{20}=100 \\ \frac{8}{20} x=100 \\ x=250 \end{gathered}$ <br> Or $\begin{aligned} & E(x)=\frac{1}{20}(-7000-x) \\ & \quad+\frac{1}{10}(-5000-x) \\ & +\frac{1}{4}(-1000-x)+\frac{3}{5}(2000)=0 \\ & -7000-x-10000-2 x-5000-5 x \\ & +24000=0 \\ & 2000=8 x \Rightarrow 250=x \end{aligned}$ | Scale 10D (0, 3, 5, 8, 10) <br> Low Partial Credit: <br> Any relevant use of $x$, excluding $(9000+x)$ <br> Mid Partial Credit: <br> $E(x)$ fully formulated (LHS). <br> $\left(1900+\frac{8}{20} x\right)$ or equivalent and stops. $\frac{x}{20}+\frac{x}{10}+\frac{x}{4}$ <br> High Partial Credit <br> Relevant equation in $x$ <br> Low Partial Credit: <br> Any relevant use of $x$ e.g. $(-7000+x)$ <br> Mid Partial Credit: <br> $E(x)$ fully formulated (LHS). <br> $\left(100-\frac{8}{20} x\right)$ or equivalent and stops. <br> High Partial Credit <br> Relevant equation in $x$ |
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