Question 3 **(25 marks)**

(a) A security code consists of six digits chosen at random from the digits 0 to 9.
The code may begin with zero and digits may be repeated.
For example

0	7	1	7	3	7

is a valid code.

(i) Find how many of the possible codes will end with a zero.

(ii) Find how many of the possible codes will contain the digits 2 0 1 8 together and in this order.

(b) Find a, b, c, and d, if $\dfrac{(n+3)!\,(n+2)!}{(n+1)!(n+1)!} = an^3 + bn^2 + cn + d$, where a, b, c, and $d \in \mathbb{N}$.

Q3	Model Solution – 25 Marks	Marking Notes
(a) (i)	$10^5 \times 1$ or $100\ 000$	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Some use of 10. Identifies that 5 other digits are required to complete code. *High Partial Credit* 9^5 or equivalent 10^6
(a) (ii)	$1 \times 10 \times 10 + 10 \times 1 \times 10 + 10 \times 10 \times 1$ $3 \times 10 \times 10$ or 3×10^2 or 300	**Scale 5B (0, 2, 5)** *Partial Credit:* 10×10
(b)	$\dfrac{(n+3)!\ (n+2)!}{(n+1)!\,(n+1)!} =$ $(n+3)(n+2)(n+2) =$ $n^3 + 7n^2 + 16n + 12$ **Or** $\dfrac{(n+3)!\,(n+2)!}{(n+1)!\,(n+1)!} = an^3 + bn^2 + cn + d$ $n = 0 \rightarrow \dfrac{3!\,.2!}{1!\,1!} = 12 = d$ $n = 1 \rightarrow a + b + c + d = 36$ $n = 2 \rightarrow 8a + 4b + 2c + d = 80$ $n = 3 \rightarrow 27a + 9b + 3c + d = 150$ Solving the simultaneous equations $a = 1, b = 7, c = 16, d = 12$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Factorial expansion (e.g. $(n+3)! = (n+3)(n+2)(n+1) \ldots \ldots \ldots .1)$ Effort at a numerical value for *n* on both LHS and RHS (method 2) *High Partial Credit:* $(n+3)(n+2)(n+2)$ Four simultaneous equations