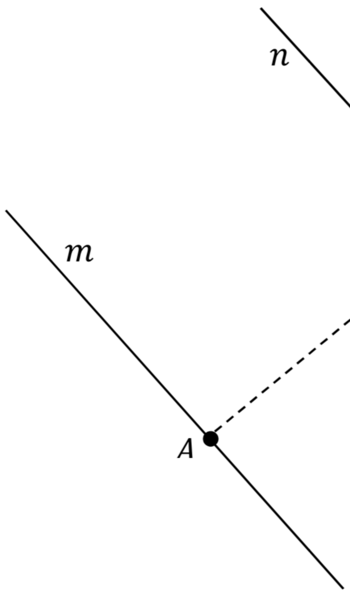
Question 5 **(25 marks)**

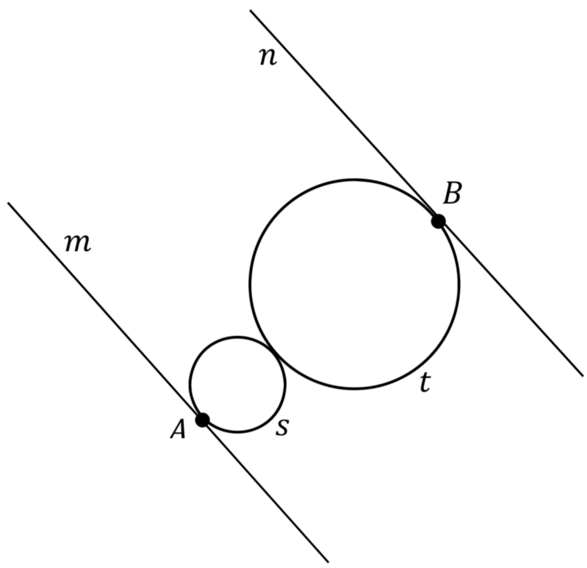
The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

(b) Find the coordinates of B, the point on the line n closest to A, as shown below.



(c) Two touching circles, s and t, are shown in the diagram. m is a tangent to s at A and n is a tangent to t at B. The ratio of the radius of s to the radius of t is $1 : 3$.
Find the equation of s.



Q5	Model Solution – 25 Marks	Marking Notes
(a)	2(-2) + 3(1) + 1 = 0 or $-4+3+1=0$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* Substitution for x **or** y in equation of line *High Partial Credit:* Substitution for x **and** y in eq. of line (LHS when no indication of 0)
(b)	Slope of *m* or *n* = $\frac{-2}{3}$ Slope of *AB* is $\frac{3}{2}$ and (−2, 1) is on *AB* $y-1=\frac{3}{2}\left(x-\ (-2)\right)$ equation of *AB* is $3x-2y+8=0$ Solve for (x,y) between $3x-2y+8=0$ and $2x+3y-51=0$ $n\cap AB=(6,13)=B$ **Or** coordinates of $B(x,y)$ $\|AB\|=\sqrt{(x+2)^2+(y-1)^2}$ Perp. distance (−2, 1) to $2x+3y-51=0$ $\left\|\frac{-4+3-51}{\sqrt{13}}\right\|=\frac{52}{\sqrt{13}}=4\sqrt{13}$ $\therefore (x+2)^2+(y-1)^2=\ (4\sqrt{13})^2$ Substituting $x=\frac{1}{2}(-3y+51)$ $(\frac{-3y+55}{2})^2+(y-1)^2=\ (4\sqrt{13})^2$ $13y^2-338y+2197=0$ $y^2-26y+169=0$ $(y-13)^2=0\rightarrow y=13$ $n\cap AB=(6,13)=B$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Slope of *AB* Equation of line formula with some substitution *Mid Partial Credit:* Equation of *AB* *High Partial Credit:* Effort at finding intersection of lines **Note**: Point of intersection, found correctly, of *n* and a relevant *AB* (with errors) merits Mid Partial Credit at least. <u>Method 2</u> *Low Partial Credit:* Perpendicular distance formula with some substitution Distance formula with some substitution *Mid Partial Credit:* Quadratic equation in x and y *High Partial Credit:* Quadratic equation in either x or y

<table>
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<td>(c)</td>
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$\overrightarrow{AB} = x$ up 8 and y up 12

Centre of s is $\frac{1}{8}(8) - 2 = -1 = h$

and $\frac{1}{8}(12) + 1 = 2{\cdot}5 = k$

Eqn s: $(x+1)^2 + (y - 2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$

Or

$s \cap t$
$\left(\frac{3(-2)+1(6)}{3+1}, \frac{3(1)+1(13)}{3+1}\right) = (0, 4)$
Centre s: $\left(\frac{0-2}{2}, \frac{4+1}{2}\right) = (-1,\ 2{\cdot}5)$
Radius : distance $(-1, 2{\cdot}5)$ to either $(-2, 1)$or $(0, 4)$ or calculated otherwise $\sqrt{3{\cdot}25}$ or $\frac{\sqrt{13}}{2}$
$(x+1)^2 + (y - 2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$

Or

using ratio 1 : 7 centre s:
$\left(\frac{1(6)+7(-2)}{1+7}, \frac{1(13)+7(1)}{1+7}\right) = (-1, 2{\cdot}5)$
Radius as above or $\mathbf{\frac{1}{8}}|AB|$= $\mathbf{\frac{\sqrt{13}}{2}}$

$(x+1)^2 + (y - 2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$
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Scale 5C (0, 2, 4, 5)
Low Partial Credit:
8 up or 12 up
Indication $4\sqrt{13}$ from(b) of relevance

High Partial Credit:
Centre and radius of circle

Low Partial Credit:
Some relevant use of 1 : 3
Midpoint of AB found once but no further work of relevance
Formula with some relevant substitution

High Partial Credit:
Centre and radius of circle

Low Partial Credit:
Some relevant use of 1 : 7
Formula with some relevant substitution

High Partial Credit:
Centre and radius of circle
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