Question 6 **(25 marks)**

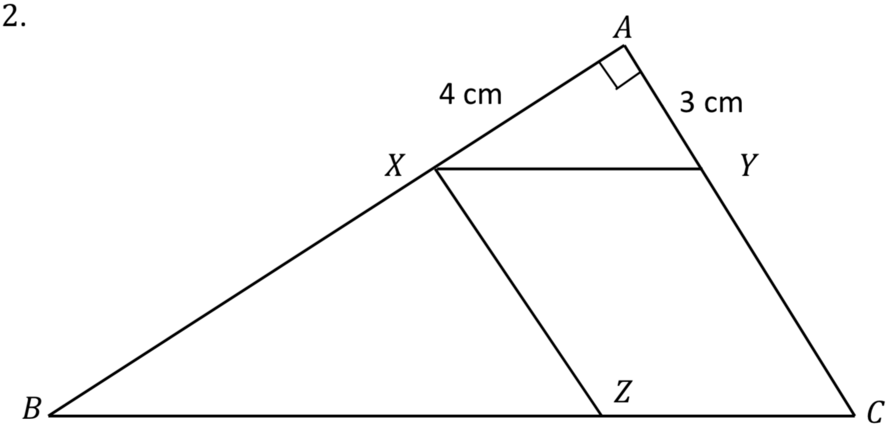
(a) Let ΔABC be a triangle. Prove that if a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, where $s, t \in \mathbb{N}$, then it also cuts $[AC]$ in the same ratio.

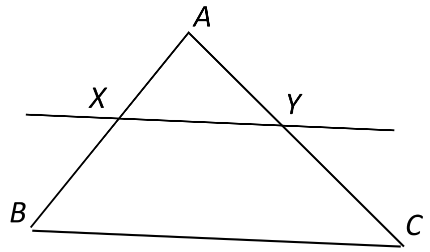
(b) In the triangle ABC shown below:

$|\angle CAB| = 90°$, $|AX| = 4$ cm, $|AY| = 3$ cm, $XY \parallel BC$, $XZ \parallel AC$,

and $|AX| : |XB| = 1 : 2$.

Find $|BZ|$.



Q6	Model Solution – 25 Marks	Marking Notes
(a)	*Diagram*:  Given: A triangle *ABC* and a line *XY* parallel to *BC* which cuts *AB* in the ratio $s : t$ where $s, t \in \mathbb{N}$. *To Prove*: $[AY] : [YC] = s : t$ *Construction*: Divide [*AB*] into $s + t$ equal parts, s of them lying along [*AX*] and t of them lying along [*XB*]. Through each point of division draw a line parallel to [*BC*] *Proof*: By a previous theorem the parallel lines cut off segments of equal length along [*AC*]. Therefore [*AC*] is divided into $s + t$ equal parts with s of them forming [*AY*] and t of them forming [*YC*]. Let k be the length of one segment on [*AC*]. $[AY] : [YC] = ks : kt \ = s : t$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Relevant diagram drawn *Mid Partial Credit:* Construction clearly indicated *High Partial Credit:* Proof missing 1 relevant step

(b)	$|XY| = \sqrt{4^2 + 3^2} = 5$ $|ZC| = 5$ $|BZ| = 10\text{cm}$ **Or** $\frac{8}{4}$ or $\frac{2}{1} = \frac{|BZ|}{5} \rightarrow |BZ| = 10\text{cm}$ **Or** $\frac{4}{12} = \frac{5}{5 + |BZ|}$ $4|BZ| + 20 = 60 \rightarrow |BZ| = 10 \text{ cm}$ Similarly: $\frac{3}{9} = \frac{5}{5+|BZ|}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $|XY|$ or $|BX|$ or $|CY|$ found Pythagoras with some substitution *High Partial Credit:* $|ZC|$ or $|BC|$ found Ratios formulated with $|BZ|$ the sole unknown