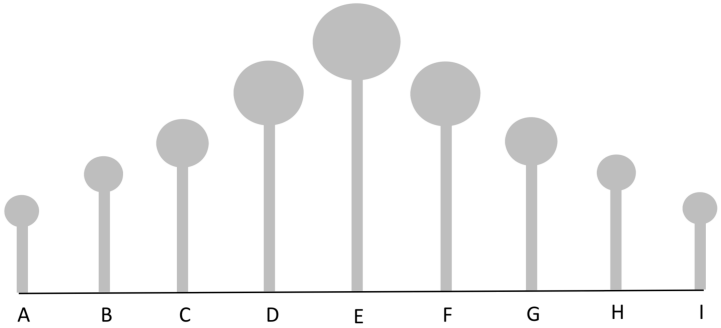
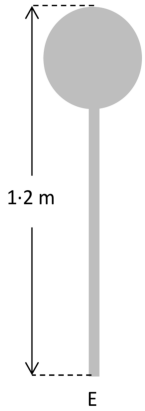
Question 7 **(50 marks)**

A section of a garden railing is shown below. This section consists of nine cylindrical bars, labelled A to I, with a solid sphere attached to the centre of the top of each bar.
The **volume** of each sphere from B to E is 1·75 times the volume of the previous sphere.



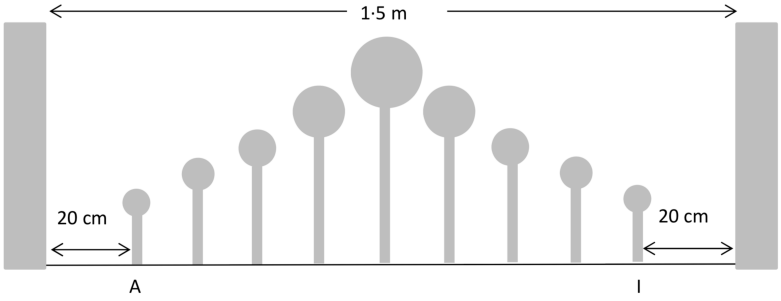
(a) The radius of sphere A is 3 cm. Find the **sum** of the volumes of the five spheres A, B, C, D, and E. Give your answer correct to the nearest cm^3.

(b) **(i)** The **surface area** of sphere E can be taken to be 503 cm^2.
The height of the railing at E (i.e. the sum of the heights of bar E and sphere E) is 1·2 metres.
Find the height of bar E, in cm, correct to 1 decimal place.

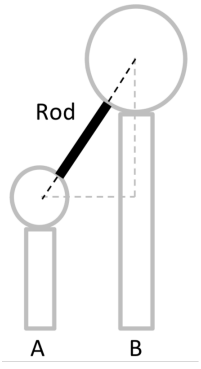


(ii) The radius of each bar is 1 cm. The volume of bar A is $71{\cdot}3\pi$ cm^3.
The heights of the bars A, B, C, D, and E form an arithmetic sequence.
Find, in cm, the height of each bar.

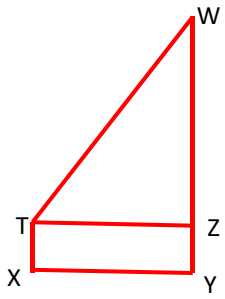
(c) There is a wall on each side of the section of railing, as shown in the diagram below which is not to drawn to scale. The distance from wall to wall is 1·5 m. The distance from the wall to bar A is 20 cm and similarly from the other wall to bar I is 20 cm.
The radius of each bar is 1 cm. The gap between each bar is identical.
Find the size of this gap.



(d) The sphere on bar A and the sphere on bar B are to be joined by a straight rod as shown in the diagram which is not to drawn to scale. Find the length of the shortest rod that will join sphere A to sphere B.
Give your answer in cm, correct to 1 decimal place.



Q7	Model Solution – 50 Marks	Marking Notes
(a)	$V = \frac{4}{3}\pi 3^3 = 36\pi = 113{\cdot}1$ $\frac{113{\cdot}1(1-1{\cdot}75^5)}{1-1{\cdot}75} = 2324{\cdot}29$ $= 2324$ **or** Volume A = $113{\cdot}1$ Volume B = $197{\cdot}925$ Volume C = $346{\cdot}36875$ Volume D = $606{\cdot}1453125$ Volume E = $1060{\cdot}754296875$ Total: $2324{\cdot}293359375$= 2324	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Volume formula with some substitution *Mid Partial Credit:* Volume of 2 spheres GP formula with some substitution *High Partial Credit:* Volume of 5 spheres G P formula fully substituted
(b) **(i)**	$4\pi r^2 = 503 \Rightarrow r = \sqrt{\frac{503}{4\pi}} = 6{\cdot}33$ Height =$120 - 2(6{\cdot}33) = 107{\cdot}3$ **Or** $\frac{4}{3}\pi r^3 = 1060{\cdot}754$ from(a) $r = 6{\cdot}326$ Height : $120 - 2(6{\cdot}326) = 107{\cdot}348$=$107{\cdot}3$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $4\pi r^2 = 503$ $\frac{4}{3}\pi r^3 =$ volume from (a) *High Partial Credit:* *r* found
(b) **(ii)**	A: $\pi 1^2 h = 71{\cdot}3\pi \Rightarrow h = 71{\cdot}3$ Height difference: $107{\cdot}3 - 71{\cdot}3 = 36$ $\frac{36}{4} = 9$ step up in each bar. **Or** $T_5 = 71{\cdot}3 + 4d = 107{\cdot}3 \rightarrow d = 9$ Height of each bar (in cm) $71{\cdot}3,\ \ 80{\cdot}3,\ \ 89{\cdot}3,\ \ 98{\cdot}3,\ \ 107{\cdot}3$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Vol formula with some substitution $\pi r^2 h = 71{\cdot}3\pi$ *Mid Partial Credit:* Height of bar A *High Partial Credit:* Difference in height between bar A and bar E

(c)	$150 - (20 + 20 + 9(2)) = 92$ $\frac{92}{8}$ cm or 11·5 cm	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Recognises 8 equal divisions Indicates subtraction of one relevant length 9×2 *High Partial Credit:* $150 - 40 - 18$ or equivalent
(d)	$V_B = 1{\cdot}75\left(\frac{4}{3}\pi 3^3\right) = 63\pi$ $V_B = \frac{4}{3}\pi r^3 = 63\pi \Rightarrow r_b = 3{\cdot}62$ cm  $\lvert XY\rvert = 1 + 11{\cdot}5 + 1 = 13{\cdot}5$ $\lvert ZW\rvert = (9 - 3) + 3{\cdot}62 = 9{\cdot}62$ $\lvert TW\rvert = \sqrt{13{\cdot}5^2 + 9{\cdot}62^2} = 16{\cdot}576$ **Or** $\tan\angle WTZ = \frac{9{\cdot}62}{13{\cdot}5} \rightarrow \lvert\angle WTZ\rvert = 35{\cdot}459°$ $\cos\angle WTZ = \frac{13{\cdot}5}{\lvert TW\rvert} \rightarrow \lvert TW\rvert = 16{\cdot}576$ The rod is: $\lvert TW\rvert - 3 - 3{\cdot}62$ $= 16{\cdot}576 - 3 - 3{\cdot}62 = 9{\cdot}95$ $\lvert TW\rvert = 10$	**Scale 5B (0, 2, 5)** *Partial Credit:* V_B formulated with some substitution $\lvert XY\rvert$ formulated $\lvert TW\rvert$ evaluated rod $= \lvert TW\rvert - r_b - r_a$ formulated with 2 relevant values