Question 8 **(60 marks)**

Acme Confectionery makes cakes and chocolate bars.

(a) **(i)** *Acme Confectionery* has launched a new bar called *Chocolate Crunch*. The weights of these new bars are normally distributed with a mean of 4·64 g and a standard deviation of 0·12 g. A sample of 10 bars is selected at random and the mean weight of the sample is found.
Find the probability that the mean weight of the sample is between 4·6 g and 4·7 g.

(ii) A company surveyed 400 people, chosen from the population of people who had bought at least one *Chocolate Crunch* bar.
Of those surveyed, 324 of them said they liked the new bar.
Create the 95% confidence interval for the population proportion who liked the new bar.
Give your answer correct to 2 decimal places.

(b) **(i)** Put one tick into the table for each statement to indicate whether the statement is **Always True**, **Sometimes True** or **Never True.**
In the table, *n* is the size of the sample and $\hat{p}$ is the sample proportion.

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed *n* and $\hat{p}$), an increased confidence level implies a wider interval.			
2. As the value of $\hat{p}$ increases (for fixed *n*), the estimated standard error of the population proportion increases.			
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed *n*), the estimated standard error of the population proportion increases.			
4. As *n*, the number of people sampled, increases (for fixed $\hat{p}$), the estimated standard error of the population proportion increases.			

(ii) Using calculus or otherwise, find the maximum value of $\hat{p}(1-\hat{p})$.

(iii) Hence, find the largest possible value of the radius of the 95% confidence interval for a population proportion, given a random sample of size 800.

(c) *Acme Confectionery* has an employee pension plan. For an employee who qualifies for the full pension, *Acme Confectionery* will pay a sum of €20 000 on the day of retirement. It will then pay a sum on the same date each subsequent year for the next 25 years. Each year the employee is paid a sum that is 1% more than the amount paid in the previous year.
What sum of money must the company have set aside on the day of retirement in order to fund this pension? Assume an annual interest rate (AER) of 2·4%.

Q8	Model Solution – 60 Marks	Marking Notes
(a) (i)	$z_1 = \dfrac{4{\cdot}6 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = -1{\cdot}05409$ $z_2 = \dfrac{4{\cdot}7 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = 1{\cdot}581138$ $p(-1{\cdot}05 < z < 1{\cdot}58)$ $= 0{\cdot}9429 - (1 - 0{\cdot}8531)$ $= 0{\cdot}796$ or 79·6%	**Scale 20D (0, 5, 10, 15, 20)** *Low Partial Credit:* z_1 formulated with some correct substitution z_2 formulated with some correct substitution *Mid Partial Credit:* z_1 and z_2 fully substituted *High Partial Credit:* –1·05 and 1·58 or equivalents
(a) (ii)	Confidence Interval: $\hat{p} \pm 1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $0{\cdot}81 - 1{\cdot}96\sqrt{\dfrac{0{\cdot}81 \times 0{\cdot}19}{400}} \leq p$ $\leq 0{\cdot}81 + 1{\cdot}96\sqrt{\dfrac{0{\cdot}81 \times 0{\cdot}19}{400}}$ $0{\cdot}77155 \leq p \leq 0{\cdot}848445$ $0{\cdot}77 \leq p \leq 0{\cdot}85$	**Scale 15 C (0, 4, 11, 15)** *Low Partial Credit:* CI formulated with some correct substitution 1·96 $\hat{p} \pm \frac{1}{\sqrt{n}}$ incomplete or completed *High Partial Credit:* CI fully substituted

(b) (i)

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed *n* and $\hat{p}$), an increased confidence level implies a wider interval.	✓		
2. As the value of $\hat{p}$ increases (for fixed *n*), the estimated standard error of the population proportion increases.		✓	
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed *n*), the estimated standard error of the population proportion increases.	✓		
4. As *n*, the number of people sampled, increases (for fixed $\hat{p}$), the estimated standard error of the population proportion increases.			✓

Scale 5C (0, 2, 4, 5)
Low Partial Credit:
Any 1 correct

High Partial Credit:
Any 2 correct

(b) (ii)+ (iii)	$M = \hat{p} - \hat{p}^2$ $\frac{dM}{d\hat{p}} = 1 - 2\hat{p}$ $\frac{dM}{d\hat{p}} = 1 - 2\hat{p} = 0$ $\hat{p} = \frac{1}{2}$ $M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ **Or** $f(p) = \hat{p} - \hat{p}^2 = -(\hat{p}^2\ \hat{p})$ $= -[(\hat{p}^2 - \hat{p} + (-\frac{1}{2})^2) - (-\frac{1}{2})^2]$ $= \frac{1}{4} - \left(\hat{p} - \frac{1}{2}\right)^2$ $\hat{p} = \frac{1}{2} \quad M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ $1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= 1{\cdot}96\sqrt{\frac{1}{4n}} = 0{\cdot}03464 = 3{\cdot}46\%$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant calculus $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{800}}$ Effort at completing the square *Mid Partial Credit* $\hat{p} = \frac{1}{2}$ or equivalent *High Partial Credit:* Maximum value	
(c)	$20\,000 + \frac{20\,000(1{\cdot}01)}{1{\cdot}024} + \frac{20\,000(1{\cdot}01^2)}{1{\cdot}024^2} + \frac{20\,000(1{\cdot}01^3)}{1{\cdot}024^3} + \cdots + \frac{20\,000(1{\cdot}01^{25})}{1{\cdot}024^{25}}$ $20\,000\left[1 + \frac{1{\cdot}01}{1{\cdot}024} + \frac{1{\cdot}01^2}{1{\cdot}024^2} + \frac{1{\cdot}01^3}{1{\cdot}024^3} + \cdots + \frac{1{\cdot}01^{25}}{1{\cdot}024^{25}}\right]$ $a = 1,\ r = \frac{1{\cdot}01}{1{\cdot}024} = \frac{505}{512}, n = 26$ $20000\left[\frac{\left(1-\frac{505^{26}}{512^{26}}\right)}{1-\frac{505}{512}}\right] = €440\,132{\cdot}40$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* 20000(1·01) or $\frac{20000}{1{\cdot}024}$ *Mid Partial Credit:* $\frac{20\,000(1{\cdot}01)}{1{\cdot}024}$ or similar term Correctly handles inflation element or completes correctly present values element and finishes *High Partial Credit:* GP with *a*, *r* and *n* identified **Note**: Treat $n = 25$ as a misreading	