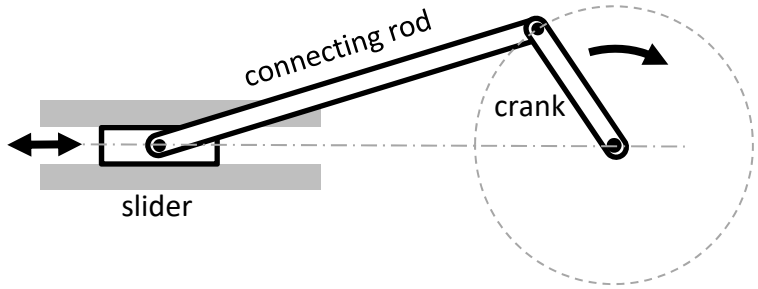
**Question 9** **(40 marks)**

In engineering, a crank-and-slider mechanism can be used to change circular motion into motion back and forth in a straight line.




In the diagrams below, the crank [*OD*] rotates about the fixed point *O*. The point *C* slides back and forth in a horizontal line. [*CD*] is the rod that connects *C* to the crank. The diagrams below show three of the possible positions for *C* and *D*. $|OD| = 10$ cm and $|DC| = 30$ cm.

**Diagram 1**
(Starting position)

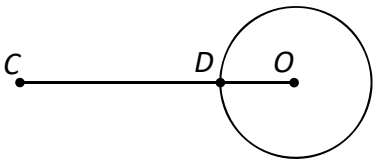



**Diagram 2**

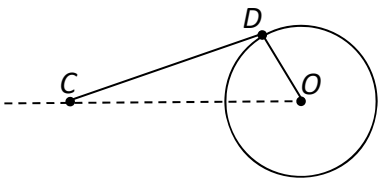
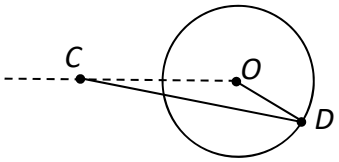
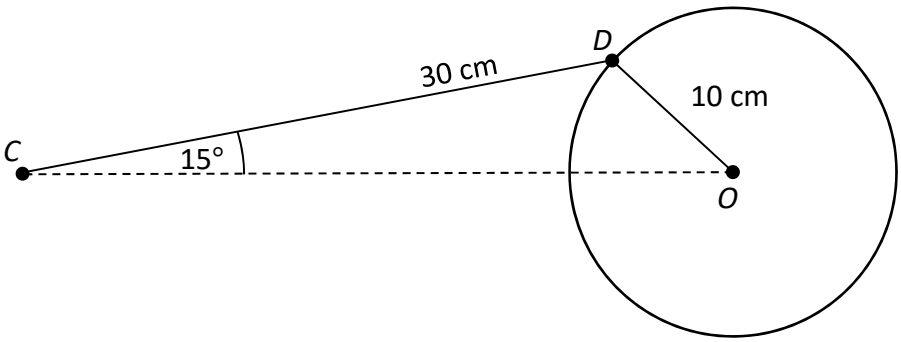



**Diagram 3**




**(a)** The diagram below shows a particular position of the mechanism with $|\angle DCO| = 15°$. Find $|\angle COD|$, correct to the nearest degree.



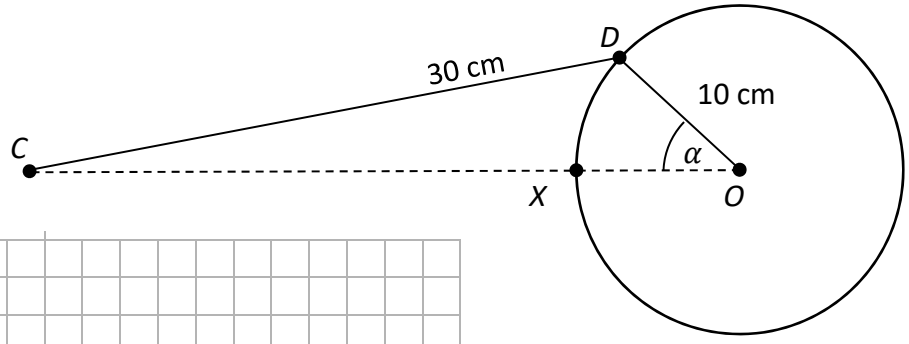
**(b)** As *D* moves in a circle around *O*, the angle $\alpha$ in the diagram below increases.
The distance |*CX*| can be considered to be a function of $\alpha$ and written as $f(\alpha)$.

**(i)** Write down the period and range of $f$.

| Period = | Range = |
|---|---|

**(ii)** Complete the table below for $f(\alpha)$.
Give your answers correct to 2 decimal places where appropriate.
(**Note: Diagram 1** at the start of this question represents $\alpha = 0°$).

| $\alpha$ | $0°$ | $90°$ | $180°$ | $270°$ | $360°$ |
|---|---|---|---|---|---|
| $f(\alpha)$ (cm) | 30 | | | | |




**(iii)** Use your values from the table to draw a rough sketch of $f$ in the domain $0° \leq \alpha \leq 360°$.

Draw your graph on this grid.

*This question continues on the next page*

**(iv)** Referring to **Diagrams 1**, **2, and 3** near the start of this question, for which of the three positions of the mechanism will a 1 degree change in $\alpha$ cause the greatest change in the position of $C$? Explain your answer.

Answer:

Explanation:

**(c)** The diagram below shows another crank-and-slider mechanism with different dimensions. In the diagram, $|AB| = 36$ cm, $|AX| = 31$ cm, and $|\angle BAO| = 10°$.
(**Note**: $|\angle OBA| \neq 90°$)
Find $r$, the length of the crank. Give your answer in cm, correct to the nearest cm.