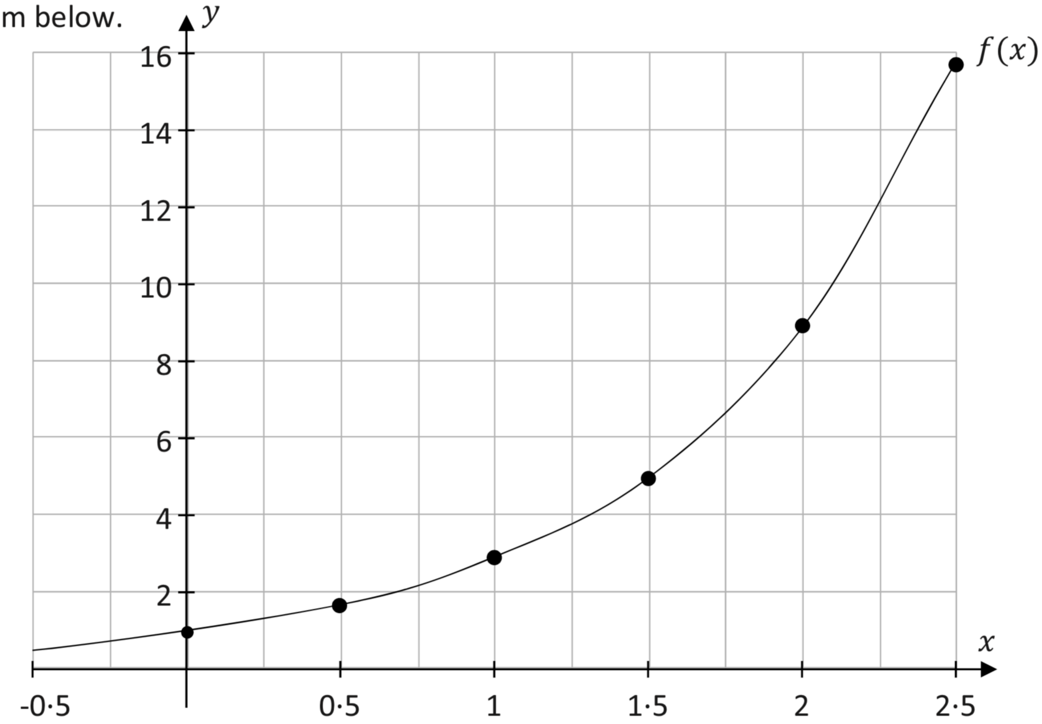
Question 2 **(25 marks)**

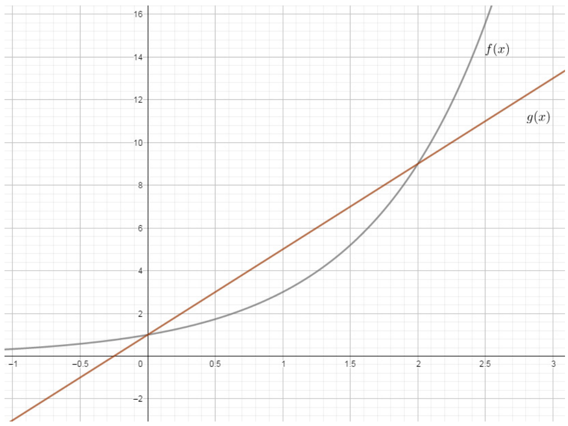
The graph of the function $f(x) = 3^x$, where $x \in \mathbb{R}$, cuts the y-axis at $(0, 1)$ as shown in the diagram below.



(a) **(i)** Draw the graph of the function $g(x) = 4x + 1$ on the diagram.

(ii) Use substitution to verify that $f(x) < g(x)$, for $x = 1{\cdot}9$.

(b) Prove, using induction, that $f(n) \geq g(n)$, where $n \geq 2$ and $n \in \mathbb{N}$.

Q2	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	(0, 1) (2, 9) 	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - 1 point on line found *High Partial Credit:* - 2 points on line found - 1 point found and plotted (apart from (0, 1) and (2, 9)) *Full Credit −1*: - Freehand graph drawn
(a) **(ii)**	$g(1{\cdot}9) = 4(1{\cdot}9) + 1 = 8{\cdot}6$ $f(1{\cdot}9) = 3^{1{\cdot}9} = 8{\cdot}06$ $f(x) < g(x)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - $g(1{\cdot}9)$ written or found - $f(1{\cdot}9)$ written or found *High Partial Credit:* - $g(1{\cdot}9)$ and $f(1{\cdot}9)$ found

<table>
<tr>
<td>(b)</td>
<td>To Prove: $3^n \geq 4n + 1$ for $n \geq 2$
$P(2): 3^2 \geq 4(2) + 1$
$9 \geq 9$, True
Assume $P(n)$ is true for $n = k$,
Now prove $P(n)$ is true for $n = k + 1$
$P(k)$: $3^k \geq 4k + 1$ for $k \geq 2$
$P(k + 1): 3^{k+1} \geq 4(k + 1) + 1$
$3^{k+1} \geq 4k + 5$

Proof: $P(k) \times 3$: $3^{k+1} \geq 3(4k + 1)$
$= 12k + 3$
$\Rightarrow$ $3^{k+1} \geq 4k + 5$
since $4k + 5 < 12k + 3$ for $k \geq 2$

True for
$n = k + 1$ provided true for $n = k$
but true for $n = 2$
$\therefore$ True for all $n \geq 2, n \in \mathbb{N}$.</td>
<td>Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
- Step $P(2)$
- $P(k)$ or $P(k + 1)$ with incorrect inequality sign

Mid Partial Credit:
- Any two of $P(2)$, $P(k)$ or $P(k + 1)$

High Partial Credit:
- Uses Step $P(k)$ to prove Step $P(k + 1)$

Full Credit −1:
- Omits conclusion but otherwise correct

<u>Note</u>: Accept Step $P(2)$, Step $P(k)$, Step $P(k + 1)$ in any order

<u>Note</u>: Accept $f(k) \geq g(k)$, $k \geq 2$ for Step $P(k)$</td>
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