Question 6 **(25 marks)**

(a) **(i)** Given that $x - \sqrt{32} = \sqrt{128} - 5x$, find the value of x, where $x \in \mathbb{R}$.
Give your answer in the form $a\sqrt{2}$, where $a \in \mathbb{N}$.

(ii) $\text{A} = \left\{\sqrt{32k^2}, \sqrt{50k^2}, \sqrt{128k^2}, \sqrt{98k^2}\right\}$, where $k \in \mathbb{N}$.

Show that the **mean** of set A is equal to the **median** of set A.

(b) Prove, using contradiction, that $\sqrt{2}$ is **not** a rational number.

<table>
<tr><th>Q6</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$x + 5x = \sqrt{128} + \sqrt{32}$
$6x = 8\sqrt{2} + 4\sqrt{2}$
$6x = 12\sqrt{2}$
$x = 2\sqrt{2}$
Or
$x - \sqrt{32} = \sqrt{128} - 5x$
$\left(x - \sqrt{32}\right)^2 = \left(\sqrt{128} - 5x\right)^2$
$\left(x - 4\sqrt{2}\right)^2 = \left(8\sqrt{2} - 5x\right)^2$
$x^2 - 8\sqrt{2}x + 32 = 128 - 80\sqrt{2}x + 25x^2$
$x^2 - 3\sqrt{2}x + 4 = 0$
$\left(x - \sqrt{2}\right)\left(x - 2\sqrt{2}\right) = 0$
$x = \sqrt{2}$ or $x = 2\sqrt{2}$
Check solutions:
$x = \sqrt{2}$
$\sqrt{2} - \sqrt{32} = \sqrt{128} - 5\sqrt{2}$
$-3\sqrt{2} = 3\sqrt{2}$ (False)
Solution: $x = 2\sqrt{2}$</td><td>Scale 10C (0, 4, 7, 10)
Low Partial Credit:
- Any relevant transposing
- $\sqrt{32}$ or $\sqrt{128}$ in the form a$\sqrt{2}$

High Partial Credit
- x term isolated in equation

Low Partial Credit:
- $\sqrt{32}$ or $\sqrt{128}$ in the form $a\sqrt{2}$
- Any relevant multiplication

High Partial Credit:
- LHS and RHS squared correctly
- Solution not in the form $a\sqrt{2}$

Full Credit –1:
- Both solutions presented

<u>Note</u>: If $\sqrt{128}$ and $\sqrt{32}$ are converted to decimals, then award low partial credit at most</td></tr>
<tr><td>(a)
(ii)</td><td>$\sqrt{32k^2}, \sqrt{128k^2}, \sqrt{98k^2}, \sqrt{50k^2}$
$4\sqrt{2}\,k,\quad 8\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 5\sqrt{2}\,k$
$4\sqrt{2}\,k,\quad 5\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 8\sqrt{2}\,k$
$\text{Mean} = \frac{24\sqrt{2}\,k}{4} = 6\sqrt{2}\,k$
$\text{Median} = 6\sqrt{2}\,k$</td><td>Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- List in ascending or descending order
- Any term written in the form $a\sqrt{2}k$ or in the form $a\sqrt{2k^2}$

High Partial Credit:
- Mean or median found
- Verified for a particular value of k

<u>Note</u>: If decimals are used then award low partial credit at most</td></tr>
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(b)	Assume $\sqrt{2}$ is rational i.e. $\sqrt{2} = \frac{p}{q}$ where p and q have no common factors (simplest form) $\Rightarrow 2 = \frac{p^2}{q^2}$ $\Rightarrow 2q^2 = p^2$ $\Rightarrow p^2$ is even $\Rightarrow$ p is even $\Rightarrow p = 2k$ for some $k \in \mathbb{Z}$ $2q^2 = p^2$ becomes $2q^2 = 4k^2$ $\Rightarrow q^2 = 2k^2$ $\Rightarrow q^2$ is even $\Rightarrow$ q is even $\Rightarrow q = 2m$ for some $m \in \mathbb{Z}$ $\therefore \sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$ $\Rightarrow$ common factor of 2 (contradiction) $\therefore \sqrt{2}$ cannot be rational.	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - $\sqrt{2} = \frac{p}{q}$ or similar *Mid Partial Credit* - deduces that p is even or equivalent - $p = 2k$ or equivalent deduced - $p^2 = 2q^2$ *High Partial Credit:* - $q = 2m$ or equivalent deduced