Question 8 **(50 marks)**

The weekly revenue produced by a company manufacturing air conditioning units is seasonal. The revenue (in euro) can be approximated by the function:

$$r(t) = 22\,500\cos\left(\frac{\pi}{26}t\right) + 37\,500, \qquad t \geq 0$$

where t is the number of weeks measured from the beginning of July and $\left(\frac{\pi}{26}t\right)$ is in radians.

(a) Find the approximate revenue produced 20 weeks after the beginning of July. Give your answer correct to the nearest euro.

(b) Find the two values of the time t, within the first 52 weeks, when the revenue is approximately €26 250.

(c) Find $r'(t)$, the derivative of $r(t) = 22\,500\cos\left(\frac{\pi}{26}t\right) + 37\,500$.

(d) Use calculus to show that the revenue is increasing 30 weeks after the beginning of July.

(e) Find a value for the time t, within the first 52 weeks, when the revenue is at a minimum. Use $r''(t)$, to verify your answer.

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$r(20) = 22500\cos\left(\frac{\pi}{26}(20)\right) + 37500$ $= 22500\cos\left(\frac{20\pi}{26}\right) + 37500$ $= €20658{\cdot}51$ $\approx €20659$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Any relevant substitution - $r(20)$ or $t = 20$ *High Partial Credit:* - Correct substitution *Full Credit −1:* - Uses degrees as unit of measurement, giving an answer of €59980
(b)	$22500\cos\left(\frac{\pi}{26}t\right) + 37500 = 26250$ $22500\cos\left(\frac{\pi}{26}t\right) = -11250$ $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ $\frac{\pi}{26}t = \frac{2\pi}{3}$ and $\frac{\pi}{26}t = \frac{4\pi}{3}$ $t = \frac{52}{3}$ and $t = \frac{104}{3}$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Equation formed - Trial and improvement with at least two values tested *Mid Partial Credit:* - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{11250}{22500}$ *High Partial Credit:* - 1 correct solution to equation found

(c)	$r'(t) = 22500\left[-\sin\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $= -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}t)\right]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant differentiation *High Partial Credit:* - Chain rule applied
(d)	$r'(30) = -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}(30))\right]$ $= 402{\cdot}164\pi$ $= 1263{\cdot}44$ > 0 ⇒ Increasing	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Some relevant substitution into answer from (c) - $r'(t) > 0$ - $\frac{dy}{dx} > 0$ *High Partial Credit:* - $r'(30)$ found but no conclusion or incorrect conclusion *Note*: If calculus is not used then award no credit for the solution

(e)	$-\frac{11250}{13}\pi\left[\sin\left(\frac{\pi}{26}t\right)\right]=0$ $\sin\left(\frac{\pi}{26}t\right)=0$ $\frac{\pi}{26}t=0$ and $\frac{\pi}{26}t=\pi$ $t=0$ and $t=26$ $r''(t)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $t=0$: $r''(t)<0$ $\Rightarrow$ Max $t=26$: $r''(t)>0$ $\Rightarrow$ Min Or Range: $[37500-22500, 37500+22500]$ $=[15{,}000, 60{,}000]$ $22500\cos\left(\frac{\pi}{26}t\right)+37500=15000$ $22500\cos\left(\frac{\pi}{26}t\right)=15000-37500$ $22500\cos\left(\frac{\pi}{26}t\right)=-22500$ $\cos\left(\frac{\pi}{26}t\right)=-1$ $\frac{\pi}{26}t=\pi$ $\therefore t=26$ $r''(26)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26})26\right)\right]\left(\frac{\pi}{26}\right)$ >0 $\Rightarrow$ Min	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - $r'(t)=0$ - $\frac{dy}{dx}=0$ - States $r''(t)>0$ at minimum value - $t=26$ and no further work *Mid Partial Credit* - $t=0$ or $t=26$ found with supporting work - $r''(t)$ found *High Partial Credit:* - $t=26$ found with supporting work and $r''(t)$ found (including use of chain rule)