Question 7   **(45 marks)**

The closed line segment [0, 1] is shown below. The first three steps in the construction of the *Cantor Set* are also shown:

- Step 1 removes the open middle third of the line segment [0, 1] leaving two **closed line segments** (i.e. the end points of the segments remain in the *Cantor Set*)
- Step 2 removes the middle third of the two remaining segments leaving four closed line segments
- Step 3 removes the middle third of the four remaining segments leaving eight closed line segments.

The process continues **indefinitely**. The set of points in the line segment [0, 1] that are **not** removed during the process is the *Cantor Set.*

(a)   **(i)**   Complete the table below to show the length of the line segment(s) removed **at each step** for the first 5 steps. Give your answers as fractions.

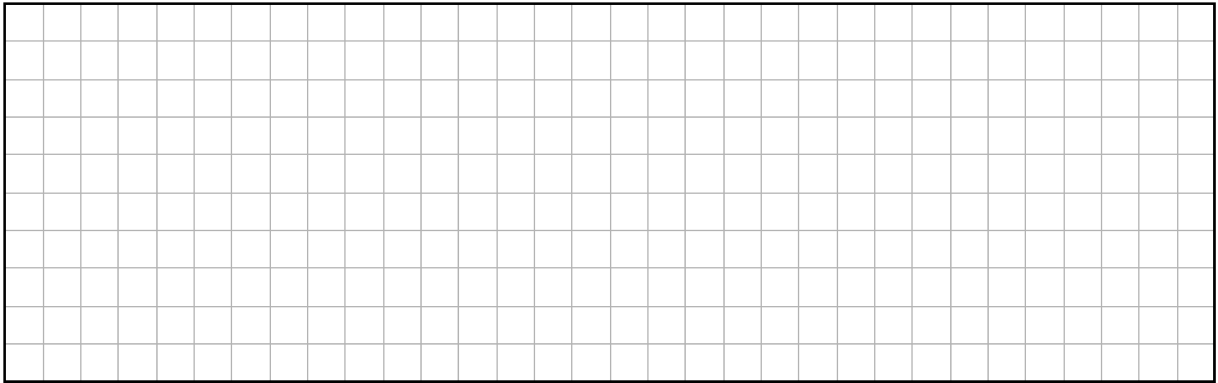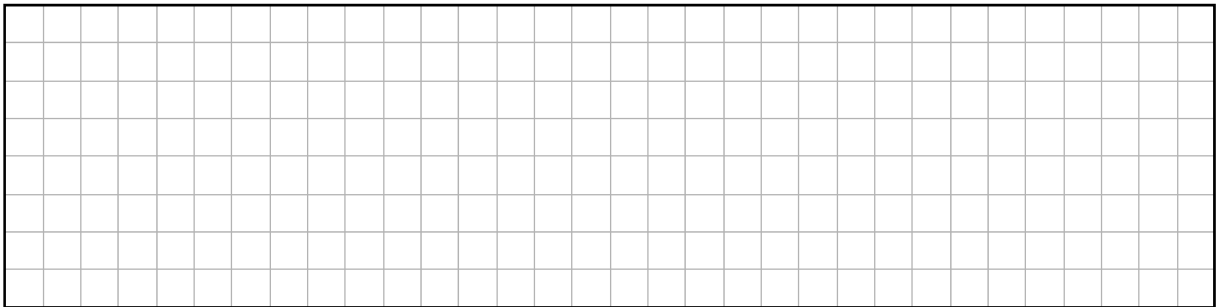| Step	Step1	Step 2	Step 3	Step 4	Step 5
Length Removed	$\dfrac{1}{3}$	$\dfrac{2}{9}$			

(ii) Find the **total** length of all of the line segments removed from the initial line segment of length 1 unit, after a finite number (*n*) of steps in the process.
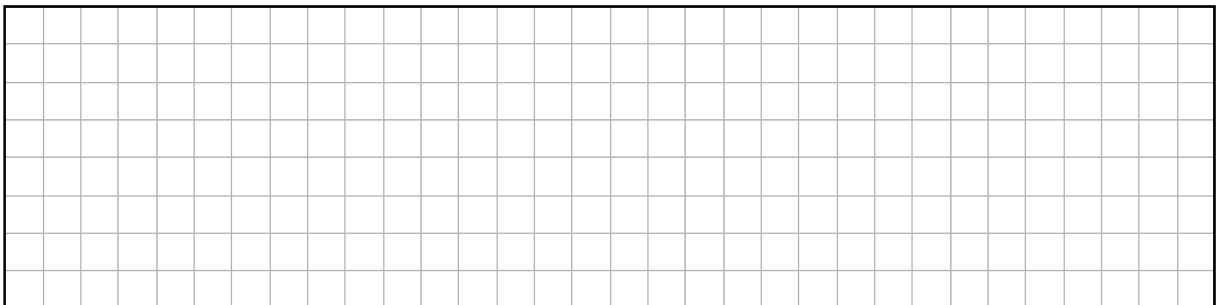Give your answer in terms of *n*.

(iii) Find the total length removed, from the initial line segment, after an infinite number of steps of the process.
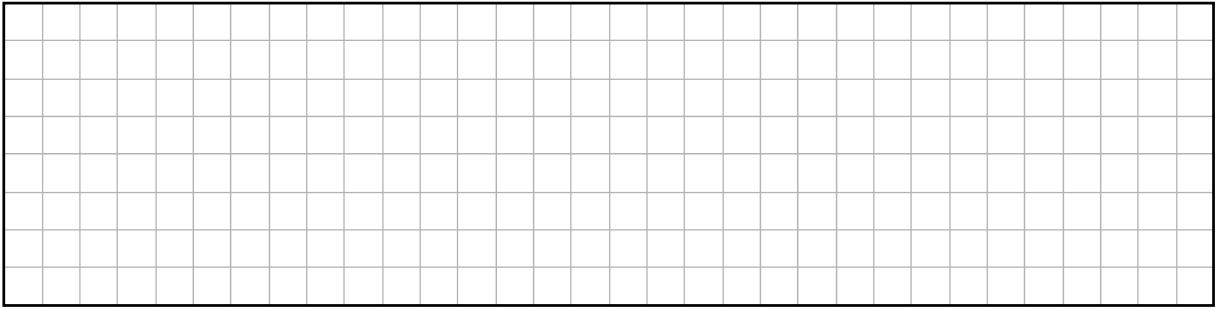
(b) **(i)** Complete the table below to identify the end-points labelled in the diagram.
Give your answers as **fractions.**

Label	A	B	C	D	E	F
End-point						

This question continues on the next page.

(ii) Give a reason why $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81}$ is a point in the *Cantor Set*.

(iii) The limit of the series $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$ is a point in the *Cantor Set*. Find this point.