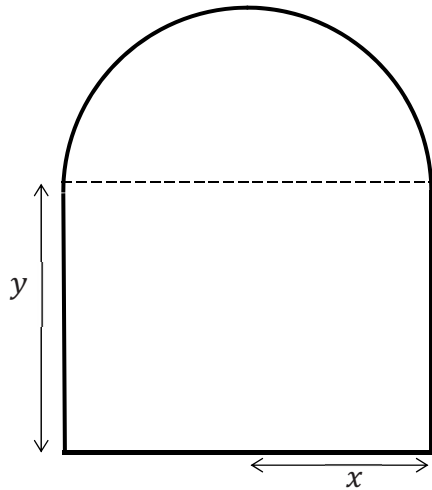


Question 9

(55 marks)



Photograph by Lionel Wall.
http://greatenglishchurches.co.uk/html/castle_rising/html

Norman windows consist of a rectangle topped by a semi-circle as shown above. Let the height **of the rectangle** be y metres and the radius of the semi-circle be x metres as shown. The perimeter of the window is P .

(a) (i) Write P in terms of x , y , and π .

(ii) In a particular Norman window the perimeter $P = 12$ metres.

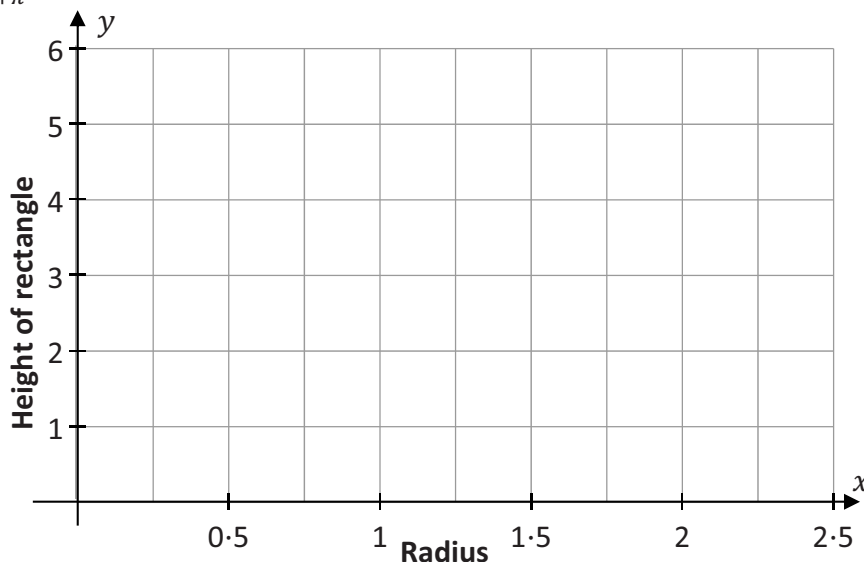
Show that $y = \frac{12 - (2 + \pi)x}{2}$ for $0 \leq x \leq \frac{12}{2 + \pi}$ where $x \in \mathbb{R}$.

(b) (i) Complete the table on the right.

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x	0	$\frac{12}{2 + \pi}$
$y = \frac{12 - (2 + \pi)x}{2}$		

(ii) On the diagram below, draw the graph of the linear function, $y = \frac{12 - (2 + \pi)x}{2}$
for $0 \leq x \leq \frac{12}{2 + \pi}$ where $x \in \mathbb{R}$.



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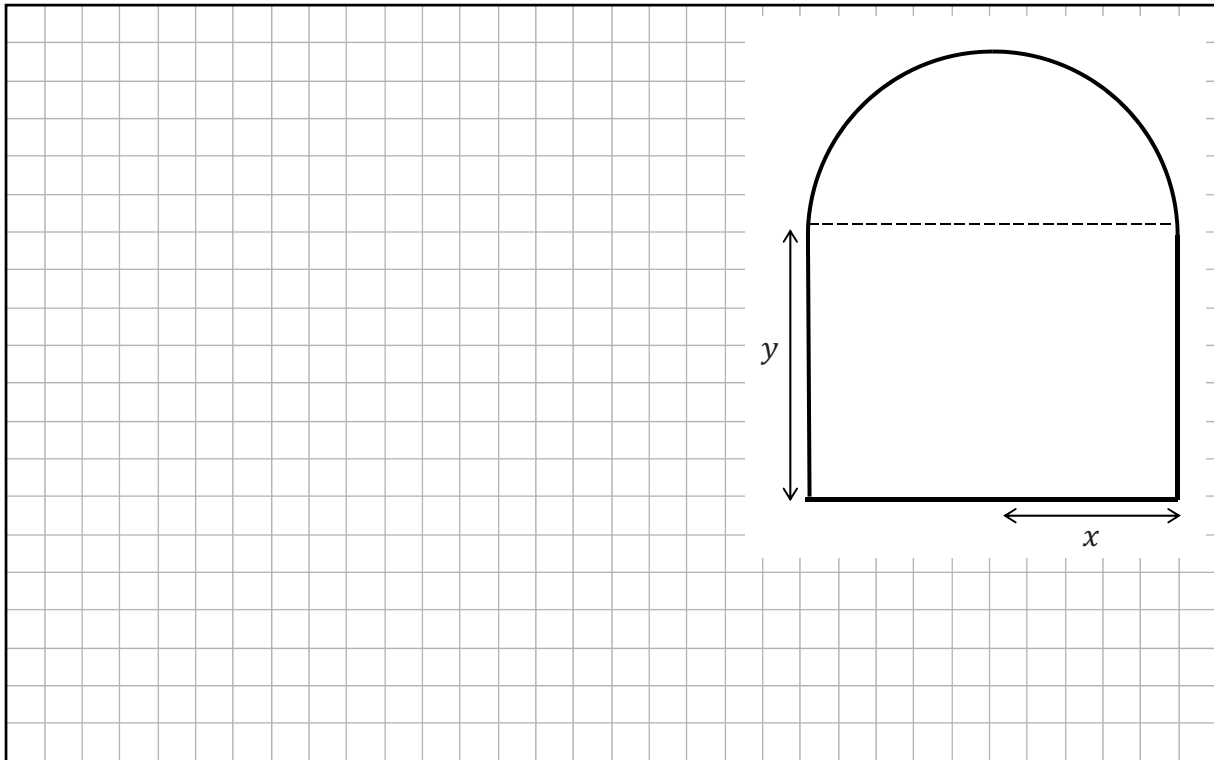
(iii) Find the slope of the graph of y , correct to 2 decimal places.
Interpret this slope in the context of the question.

Slope:	
Interpretation:	

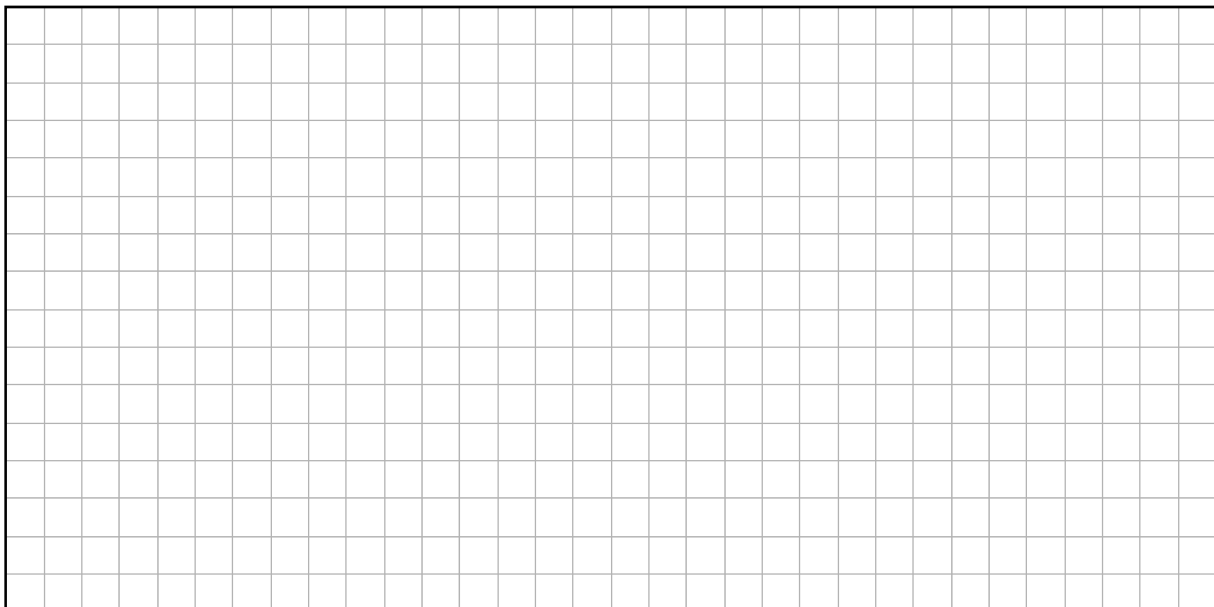
This question continues on the next page.

- (c) (i) The Norman window shown below has a perimeter of 12 metres
and $y = \frac{12 - (2 + \pi)x}{2}$.

Show that the function $a(x) = \frac{24x - (\pi + 4)x^2}{2}$ represents the area of the window, in terms of x and π .



- (ii) Find $a'(x)$.



- (iii) Find the relationship between x and y when the area of the window in **part (c)(i)** is at its maximum.

