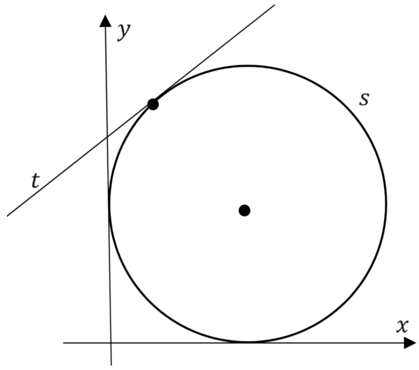
Question 3 **(25 marks)**

(a) The point $(-2, k)$ is on the circle $(x-2)^2 + (y-3)^2 = 65$.
Find the two possible values of k, where $k \in \mathbb{Z}$.

(b) The circle s is in the first quadrant.
It touches both the x-axis and the y-axis.
The line $t: 3x - 4y + 6 = 0$ is a tangent to s as shown.
Find the equation of s.



Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(-2-2)^2+(k-3)^2=65$ $16+(k-3)^2=65$ $(k-3)^2=49$ $k-3=\pm\sqrt{49}=\pm 7$ $k=10$ and $k=-4$ Or $k^2-6k+9=49$ $k^2-6k-40=0$ $(k-10)(k+4)=0$ $k=10$ and $k=-4$ Or $x^2-4x+4+y^2-6y+9=65$ $x^2+y^2-4x-6y=52$ $4+k^2+8-6k=52$ $k^2-6k-40=0$ $(k-10)(k+4)=0, \therefore k=10,\ k=-4$ Or Centre (2, 3) , radius $\sqrt{65}$ $\sqrt{(2+2)^2+(3-k)^2}=\sqrt{65}$ and proceed as above	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution Centre or radius *High Partial Credit:* Equation in k^2

(b)		
	Both axes are tangents to the circle. centre (-g,-g) and radius is g Perpendicular distance (-g,-g) to $3x - 4y + 6 = 0$ is equal to the radius $$\frac{-3g + 4g + 6}{5} = -g$$ $$g + 6 = \pm 5g$$ $g + 6 = -5g, \therefore\ -g = 1$ Centre (1, 1) and radius 1 Equation: $(x-1)^2 + (y-1)^2 = 1$ or $x^2 + y^2 - 2x - 2y + 1 = 0$ Or s is a tangent to both axes therefore $c = g^2 = f^2$ So equation is in the form $x^2 + y^2 + 2gx + 2gy + g^2 = 0$ $3x - 4y + 6 = 0\ => y = \frac{3x+6}{4}$ Substitute into circle: $x^2 + \left(\frac{3x+6}{4}\right)^2 + 2gx + \frac{2g(3x+6)}{4} + g^2 = 0$ => $25x^2 + x(36 + 56g) + 36 + 48g + 16g^2 = 0$ Tangent therefore $b^2 = 4ac$ $(36 + 56g)^2 = 4(25)(36 + 48g + 16g^2)$ $2g^2 - g - 3 = 0$ $g = -1$ and $g = \frac{3}{2}$ But can't have positive g as the co-ordinate $-g$ is in first quadrant. => $g = -1$. Therefore equation is $x^2 + y^2 - 2x - 2y + 1 = 0$ or $(x-1)^2 + (y-1)^2 = 1$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* centre *(-g,-g)* or equivalent *Mid Partial Credit:* Substitution into perpendicular distance formula completed Perpendicular distance of centre to tangent equals radius with some substitution *High Partial Credit:* equation in g or equivalent *Low Partial Credit:* $c = g^2$ or f^2 Effort to express $\boldsymbol{x}$ in terms of $\boldsymbol{y}$ or equivalent *Mid Partial Credit:* Substitution into circle equation completed *High Partial Credit:* Quadratic equation in g or f