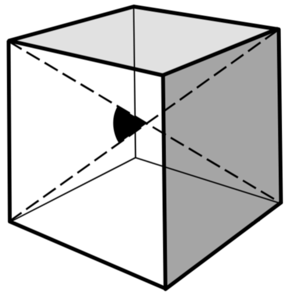
Question 4 **(25 marks)**

(a) Show that $\cos 2\theta = 1 - 2\sin^2\theta$.

(b) Find the cosine of the acute angle between two diagonals of a cube.



Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1-\sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$ Or Taking RHS $1-2\sin^2 A = 1 - 2(1-\cos^2 A)$ $= -1 + 2\cos^2 A$ $= -(\cos^2 A + \sin^2 A) + 2\cos^2 A$ $= \cos^2 A - \sin^2 A$ $= \cos A\cos A - \sin A\sin A = \cos 2A$ Or $(\cos A + i\sin A)^2 = \cos 2A + i\sin 2A$ $(\cos A + i\sin A)^2$ $= \cos^2 A + 2i\sin A\cos A + (i\sin A)^2$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1-\sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\cos(A+B)$ formula with some substitution $\cos^2 A + \sin^2 A = 1$ indicated or clearly implied *High Partial Credit:* $\cos 2A = \cos^2 A - \sin^2 A$ *Low Partial Credit:* $\cos^2 A + \sin^2 A = 1$ indicated or clearly implied $(\cos A + i\sin A)^2$ expanded *High Partial Credit:* $\cos 2A = \cos^2 A - \sin^2 A$

(b)	Let length of side be x Diagonal of any face $= \sqrt{x^2 + x^2} = \sqrt{2}x$ Internal diagonal $= x^2 + \left(\sqrt{2}x\right)^2 = \sqrt{3}x$ By cosine rule: $x^2 = \left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - 2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$ $\cos A = \dfrac{\left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$ $\cos A = \frac{1}{3}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* Fully substituted cosine rule **Note:** Accept and mark work where a consistent numerical value is assigned to one side of the cube.
	Or Drop perpendicular from intersecting diagonals to side of cube, thereby creating angle $A/2$ at vertex in a right-angled triangle. $\sin\frac{A}{2} = \dfrac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}} = \frac{1}{\sqrt{3}}$ $\therefore \cos\frac{A}{2} = \frac{\sqrt{2}}{\sqrt{3}}$ $\cos A = 2\cos^2\frac{A}{2} - 1 = 2\left(\frac{2}{3}\right) - 1 = \frac{1}{3}$ Also: $\sin\frac{A}{2} = \frac{1}{\sqrt{3}} \rightarrow \frac{A}{2} = 35{\cdot}2643896^\circ$ $A = 70{\cdot}5287792^\circ$ $\cos A = 0{\cdot}33236$	*Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* $\sin\frac{A}{2}$ fully substituted