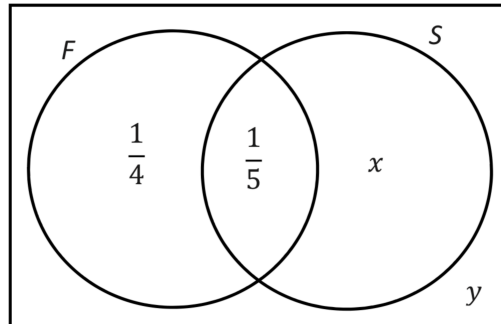


Question 6

(25 marks)

- (a) Two **independent** events F and S are represented in the Venn diagram shown below.
 $P(F \setminus S) = \frac{1}{4}$, $P(F \cap S) = \frac{1}{5}$, $P(S \setminus F) = x$, and $P(F \cup S)' = y$, where $x, y \neq 0$.
Find the value of x and the value of y .



- (b) In a club there are German, Irish and Spanish children only.
There are 10 Spanish children.
There are twice as many Irish children as German children.
They are all in a group waiting to get on a swing.
One child will be selected at random to go first and will not re-join the group.
Then a second child will be selected at random to go next.
The probability that the first child selected will be German and that the second child selected will not be German is $\frac{1}{6}$.
Find how many children are in the club.

Q6	Model Solution – 25 Marks	Marking Notes
(a)	<p>$P(F \cap S) = P(F) \times P(S)$ since the events are independent.</p> $\frac{1}{5} = \frac{9}{20} \times P(S)$ $\Rightarrow P(S) = \frac{4}{9}$ <p>So $P(S \setminus F) = \frac{4}{9} - \frac{1}{5} = \frac{11}{45} = x$</p> <p>Or</p> $P(S) = \frac{1}{5} + x$ $\frac{1}{5} = \frac{9}{20} \left(\frac{1}{5} + x \right) \Rightarrow \frac{11}{45} = x$ <p>Or</p> $P(F S) = \frac{P(F \cap S)}{P(S)} = P(F)$ $\frac{\frac{1}{5}}{\frac{1}{5} + x} = \frac{9}{20} \Rightarrow x = \frac{11}{45}$ <p>Or</p> $P(S F) = \frac{P(S \cap F)}{P(F)} = P(S)$ $\frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5} + x \Rightarrow x = \frac{11}{45}$ $y = 1 - \frac{11}{45} - \frac{1}{5} - \frac{1}{4} = \frac{11}{36}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> $P(F \cap S) = P(F) \times P(S)$ or equivalent</p> $P(F) = \frac{1}{4} + \frac{1}{5}$ $P(S) = x + \frac{1}{5}$ $\frac{1}{4} + \frac{1}{5} + x + y = 1$ <p><i>High Partial Credit</i> x found</p>

<p>(b)</p> <p>If n Germans then $2n$ Irish and $3n+10$ children in total</p> $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$ $\frac{2n^2+10n}{9n^2+57n+90} = \frac{1}{6}$ $3n^2 + 3n - 90 = 0$ $n^2 + n - 30 = 0$ $(n+6)(n-5) = 0$ <p>$n = 5$ German children.</p> <p>There are 10 Irish (and 10 Spanish) so 25 children in the club.</p> <p>Or</p> <p>25 by trial and improvement method:</p> <p>5 German, 10 Irish, 10 Spanish and verified to indicate $\frac{5}{25} \times \frac{20}{24} = \frac{1}{6}$</p>	<p>Scale 15D (0, 5, 7, 11, 15)</p> <p><i>Low Partial Credit:</i> $2n$ $3n+10$ One correct probability e.g. $\frac{n}{3n+10}$</p> <p><i>Mid Partial Credit:</i> $\frac{n}{3n+10}$ and $\left(\frac{2n+10}{\blacksquare} \text{ or } \frac{\blacksquare}{3n+9}\right)$</p> <p><i>High Partial Credit:</i> $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$</p> <p><i>Low Partial Credit:</i> Some correct element in approach</p> <p><i>Mid Partial Credit</i> Tests more than one value</p> <p><i>High Partial Credit:</i> Correct number of each nationality but not verified that probability is $\frac{1}{6}$</p> <p>Correct answer (25) with no supporting work</p>
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