(a) A cattle feeding trough of uniform cross section and 2·5 m in length, is shown in **Figure 1**.
The front of the trough (segment *ABC*) is shown in **Figure 2**.
The front of the trough is a segment of a circle of radius 90 cm.
The height of the trough, |*DB*|, is 30 cm.
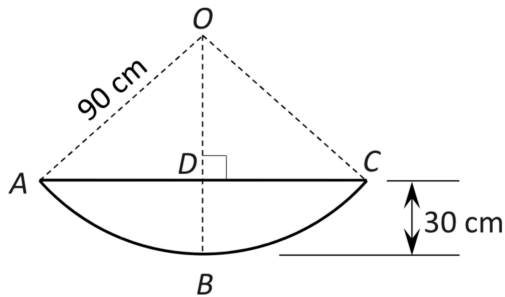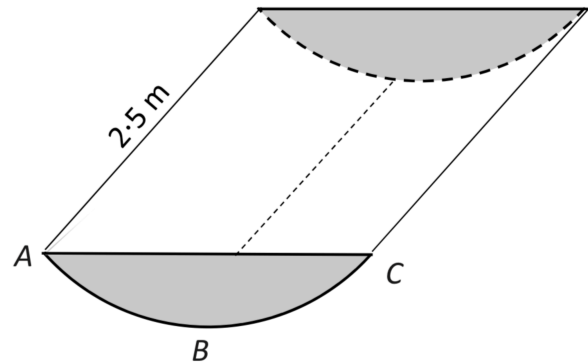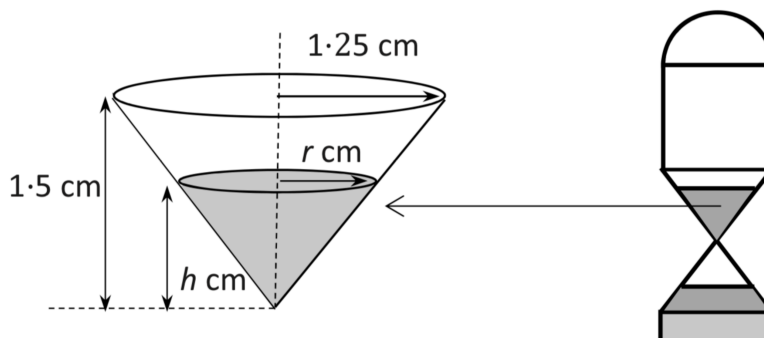|*OA*| = |*OC*| = |*OB*| = 90 cm. [*OB*] ⊥ [*AC*].

Figure 2

Figure 1

(i) Find |*AD*|. Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{Z}$.

(ii) Find |∠*DOA*|. Give your answer in radians, correct to 2 decimal places.

(iii) Find the area of the **segment** *ABC*. Give your answer **in m²** correct to 2 decimal places.

(iv) Find the **volume** of the trough. Give your answer in m³, correct to 2 decimal places.

(b) A sand timer for games is shown in the diagram.
Each half of the timer consists of a hemisphere,
a cylinder of height 3·5 cm and a cone of height 1·5 cm.
All of the parts have a radius of 1·25 cm.
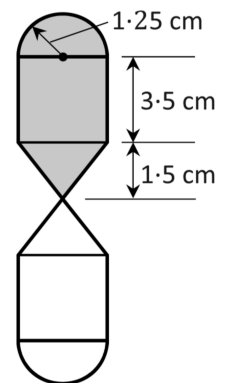
(i) The upper half of the timer is full of sand.
Find the volume of sand in the upper half of the timer.
Give your answer in cm³ correct to 2 decimal places.

(ii) Sand flows from the top half of the timer into the bottom part.
As it flows the top surfaces in both parts remain level.
At a certain time, 98% of the sand has flowed into the bottom half of the timer.
Find *h*, the height of the remaining sand (in the conical part of the top of the timer).
Give your answer in cm, correct to 2 decimal places.

Section B

Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	$$\|AD\|^2 = 90^2 - 60^2$$ $$90^2 = 60^2 + \|AD\|^2$$ $$\|AD\| = \sqrt{8100 - 3600} = \sqrt{4500} = 30\sqrt{5}$$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\|OD\| = 60$ Pythagoras formulated Effort to find angle other than $\angle ODA$ *High Partial Credit:* $\sqrt{8100 - 3600}$ or equivalent
(a) (ii)	$$\cos(\angle DOA) = \frac{60}{90}$$ $$\cos^{-1}\left(\frac{6}{9}\right) = 0{\cdot}84$$ Or $$\sin(\angle DOA) = \frac{30\sqrt{5}}{90} = \frac{\sqrt{5}}{3} = 0{\cdot}745356$$ $$\|\angle DOA\| = 48{\cdot}189°$$ $$\|\angle DOA\| = 0{\cdot}84139 = 0{\cdot}84$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Relevant trigonometric ratio formulated *High Partial Credit:* Relevant trigonometric ratio fully substituted
(a) (iii)	Area of sector: $\frac{1}{2}r^2\theta$ $\frac{1}{2}(0{\cdot}9)^2 \times 2(0{\cdot}84) = 0{\cdot}6804$ m² Area $\triangle ACO$: $\frac{1}{2}\|AC\|\|OD\| = \frac{1}{2}(60\sqrt{5})60$ cm² $\frac{1}{2}(1{\cdot}34164)(0{\cdot}6) = 0{\cdot}40$ m² Or Area $\triangle ACO$: $\frac{1}{2}\|AO\|\|OC\|\sin(\angle AOC) =$ $$\frac{1}{2}(90)(90)\sin 2(48{\cdot}189°)$$ $$= 4024{\cdot}9174 \text{ cm}^2 = 0{\cdot}40 \text{ m}^2$$ Area of segment $= 0{\cdot}6804 - 0{\cdot}40 = 0{\cdot}28$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* Formula for area of sector with some substitution Formula for area of $\triangle ACO$ with some substitution *Mid Partial Credit:* One relevant area fully substituted *High Partial Credit:* Both relevant areas fully substituted Mishandling conversion of units
(a) (iv)	Volume $= 0{\cdot}28 \times 2{\cdot}5 = 0{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Formula for **volume** of trough with some substitution Indicates some relevant use of $2{\cdot}5$ *High Partial Credit:* Formula fully substituted

(b) **(i)**	Volume $=$ $\pi\left[\left(\left(\frac{2}{3}\right)1\cdot25^3\right)\right]$ $+\pi[(1\cdot25^2\times3\cdot5)]$ $+\pi\left[\left(\left(\frac{1}{3}\right)1\cdot25^2\times1\cdot5\right)\right]$ $=\ 4\cdot0906+17\cdot1805+2\cdot4544$ $=23\cdot73$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* 1 volume formula with some substitution *Mid Partial Credit* 2 volumes fully substituted *High Partial Credit:* 3 volumes fully substituted
(b) **(ii)**	$23\cdot73\times0\ 02=0\cdot4746$ cm³ $\dfrac{r}{h}=\dfrac{1\cdot25}{1\cdot5}=\dfrac{5}{6}$ $r=\dfrac{5h}{6}$ Volume in cone $=\frac{1}{3}\pi\left(\frac{5h}{6}\right)^2\times h=0\cdot4746$ $h^3=\dfrac{0\cdot4746.3.6}{25\pi}=0\cdot65262$ $h=\sqrt[3]{0\cdot65262}\ =0\cdot8674$ $h=0\cdot87$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* volume $\times0\cdot98$ or equivalent volume multiplied by 2% effort at $r:h$ *High Partial Credit:* Volume formula expressed in one variable