Question 1 **(25 marks)**

(a) $f(x) = x^2 + 5x + p$ where $x \in \mathbb{R}$, $-3 \leq p \leq 8$, and $p \in \mathbb{Z}$.

(i) Find the value of p for which $x + 3$ is a factor of $f(x)$.

(ii) Find the value of p for which $f(x)$ has roots which differ by 3.

(iii) Find the two values of p for which the graph of $f(x)$ will not cross the x-axis.

(b) Find the range of values of x for which $|2x + 5| - 1 \leq 0$, where $x \in \mathbb{R}$.

Q1	**Model Solution – 25 Marks**	**Marking Notes**
(a) **(i)**	$f(-3) = 0$ $f(-3) = -3^2 + 5(-3) + p = 0$ $9 - 15 + p = 0$ $p = 6$ **Or** $x^2 + 5x + p = (x + 3)(x + a)$ $= x^2 + x(a + 3) + 3a$ $a + 3 = 5$ $a = 2$ $p = 3a$ $p = 6$ **Or** $x + 2$ $x + 3 \,\big)\, x^2 + 5x + p$ $x^2 + 2x$ $3x + p$ $3x + 6$ $p - 6 = 0 \quad p - 6$ $p = 6$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Demonstrates understanding of $x + 3$ as factor or – 3 as root e.g. $(x + 3)$, $f(-3)$ *High Partial Credit:* Relevant equation in p (with p as only unknown)

(a) **(ii)**	$x^2 + 5x + p = (x - \alpha)(x - \alpha - 3)$ $= x^2 + x(-\alpha - \alpha - 3) + \alpha^2 + 3\alpha$ $-2\alpha - 3 = 5$ $\alpha = -4$ $p = 16 - 12$ $p = 4$ **Or** $\alpha, \alpha + 3 =$ roots $\alpha + \alpha + 3 = -5$ $2\alpha = -8$ $\alpha = -4$ and $\alpha + 3 = -1$ $p = (-1)(-4) = 4$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Demonstrates understanding of 3 as difference of roots e.g. α with $\alpha \pm 3$ $x^2 - x(\text{sum}) + \text{product} = 0$ One correct value for p $x^2 + 5x + p > 0$ Sketch of U-shaped quadratic with turning point above the x-axis *High Partial Credit:* Relevant equation in α (with α as only unknown *Full Credit* (– 1): $p > 6{\cdot}25$
(a) **(iii)**	$b^2 - 4ac < 0$ $5^2 - 4(1)(p) < 0$ $25 - 4p < 0$ $4p > 25$ $p > 6{\cdot}25$ $p = 7$ and $p = 8$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $b^2 - 4ac$ One correct value for p $x^2 + 5x + p > 0$ *High Partial Credit:* Relevant inequality in p (with p as only unknown *Full credit* (−1): $p > 6{\cdot}25$

(b)		
	$-1 \leq 2x + 5 \leq 1$ $-6 \leq 2x \leq -4$ $-3 \leq x \leq -2$ $2x + 5 \leq 1$ $2x \leq -4$ $x \leq -2$ $-1 \leq 2x + 5$ $-6 \leq 2x$ $-3 \leq x$ $-3 \leq x \leq -2$ **Or** $(2x + 5)^2 \leq 1$ $4x^2 + 20x + 25 \leq 1$ $4x^2 + 20x + 24 \leq 0$ $x^2 + 5x + 6 \leq 0$ $(x + 2)(x + 3) \leq 0$ $x = -2, x = -3$ $-3 \leq x \leq -2$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* $(2x + 5)^2 \leq 1$ one linear inequality *Mid Partial Credit:* $-1 \leq 2x + 5 \leq 1$ Identifies both linear inequalities Quadratic inequality involving 0 *High Partial Credit:* Finding −3 and −2 in Methods 1 or 2 Roots of quadratic found $-6 \leq 2x \leq -4$ or equivalent **Note:** Accept $-3 < x < -2$