Question 2 **(25 marks)**

(a) Find the two complex numbers z_1 and z_2 that satisfy the following simultaneous equations, where $i^2 = -1$:

$$\begin{aligned} iz_1 \quad\quad &= -4 + 3i \\ 3z_1 - z_2 &= 11 + 17i. \end{aligned}$$

Write your answers in the form $a + bi$ where $a, b \in \mathbb{Z}$.

(b) The complex numbers $3 + 2i$ and $5 - i$ are the first two terms of a **geometric** sequence.

(i) Find r, the common ratio of the sequence.
Write your answer in the form $a + bi$ where $a, b \in \mathbb{Z}$.

(ii) Use de Moivre's Theorem to find T_9, the ninth term of the sequence.
Write your answer in the form $a + bi$, where $a, b \in \mathbb{Z}$.

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$iz_1 = -4 + 3i$ $i(iz_1) = i(-4 + 3i)$ $-z_1 = -4i + 3i^2$ $z_1 = 3 + 4i$ $3z_1 - z_2 = 3(3 + 4i) - z_2 = 11 + 17i$ $z_2 = 9 + 12i - 11 - 17i$ $z_2 = -2 - 5i$ **Or** $z_1 = \frac{(-4 + 3i)(-i)}{(i)(-1)}$ $z_1 = 3 + 4i$ and continues	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Either equation multiplied/divided by $\boldsymbol{i}$ *Mid Partial Credit:* z_1 found z_2 written in terms of z_1 with z_1 substituted z_1 eliminated *High Partial Credit* z_1 found and substituted into second equation z_2 found by elimination
(b) (i)	$r = \frac{T_2}{T_1} = \frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$ $r = \frac{15 - 13i - 2}{9 + 4}$ $r = \frac{13 - 13i}{13}$ $r = 1 - i$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\frac{T_2}{T_1}$ *High Partial Credit:* $\frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$
(b) (ii)	$T_9 = ar^8$ $T_9 = (3 + 2i)(1 - i)^8$ $T_9 = (3 + 2i)\left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^8$ $T_9 = (3 + 2i)(\sqrt{2})^8\left(\cos\frac{7\pi(8)}{4} + i\sin\frac{7\pi(8)}{4}\right)$ $T_9 = (3 + 2i)(16)(\cos 14\pi + i\sin 14\pi)$ $T_9 = (3 + 2i)(16)(1 + 0i)$ $T_9 = 48 + 32i$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* $T_9 = ar^8$ Any correct use of De Moivre Some use of De Moivre's Theorem on r *Mid Partial Credit:* Modulus and argument found for r *High Partial Credit:* Solution in polar form with some simplification **Note**: Accept candidates r from **(b)(i)**