

Question 3**(25 marks)**

(a) $f(x) = 6x - 5$ and $g(x) = \frac{x+5}{6}$. Investigate if $f(g(x)) = g(f(x))$.

(b) The real variables y and x are related by $y = 5x^2$.

(i) The equation $y = 5x^2$ can be rewritten in the form $\log_5 y = a + b \log_5 x$.
Find the value of a and the value of b .

(ii) Hence, or otherwise, find the real values of y for which

$$\log_5 y = 2 + \log_5 \left(\frac{126}{25} x - 1 \right).$$

| Q3 | Model Solution – 25 Marks | Marking Notes |
|-------------|--|--|
| (a) | $fg(x) = f\left(\frac{x+5}{6}\right)$ $fg(x) = 6\left(\frac{x+5}{6}\right) - 5 = x$ $gf(x) = g(6x-5)$ $gf(x) = \frac{(6x-5)+5}{6} = \frac{6x}{6} = x$ | <p>Scale 10C (0, 4, 8, 10)</p> <p><i>Low Partial Credit:</i></p> $f\left(\frac{x+5}{6}\right)$ $g(6x-5)$ <p>Particular case verification</p> <p><i>High Partial Credit:</i></p> <p>One correct composition simplified to x</p> |
| (b) (i) | $\log_5 y = \log_5 5x^2$ $\log_5 y = \log_5 5 + \log_5 x^2$ $\log_5 y = 1 + 2\log_5 x$ $a = 1 \text{ and } b = 2$ | <p>Scale 5C (0, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> $\log_5 5x^2 = \log_5 y$ $\log_5 y = \log_5 5x^2$ <p><i>High Partial Credit:</i></p> $\log_5 y = \log_5 5 + \log_5 x^2$ |
| (b) (ii) | $\log_5 y = \log_5 5x^2 = 2 + \log_5 \left(\frac{126x}{25} - 1\right)$ $\log_5 5x^2 = \log_5 \left(\frac{126x}{25} - 1\right) \times 25$ $5x^2 = 126x - 25$ $5x^2 - 126x + 25 = 0$ $(5x-1)(x-25) = 0$ $x = \frac{1}{5} \text{ or } x = 25$ $y = 5x^2 = 5\left(\frac{1}{5}\right)^2 = \frac{1}{5}$ <p>or $y = 5(25)^2 = 3125$</p> | <p>Scale 10D (0, 3, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <p>Some relevant use of laws of logs</p> <p><i>Mid Partial Credit:</i></p> <p>Quadratic equation</p> <p><i>High Partial Credit:</i></p> <p>x values found</p> <p>Note: If 2 is incorrectly (non log) dealt with then award MPC at most</p> <p>Note: If incorrect work leads to a non-quadratic equation then award MPC at most</p> |