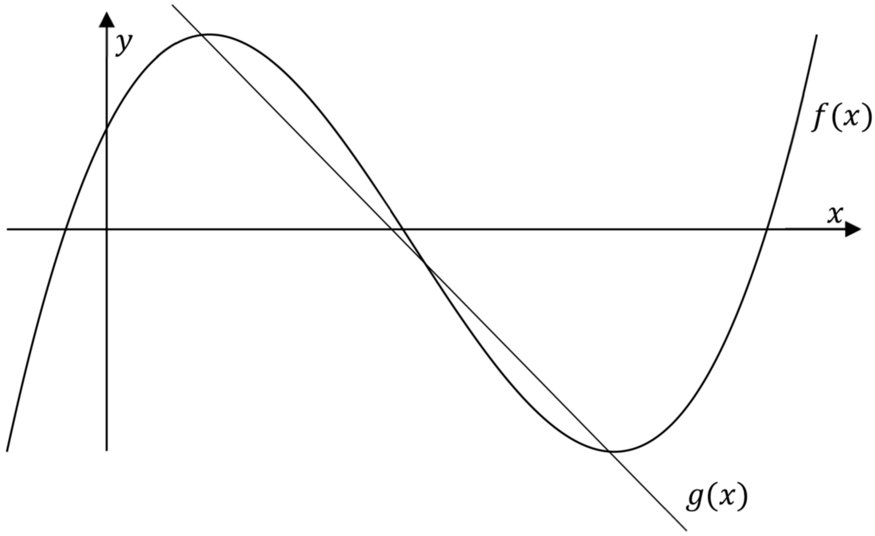
Question 4 **(25 marks)**

The diagram below shows two functions $f(x)$ and $g(x)$.

The function $f(x)$ is given by the formula $f(x) = x^3 + kx^2 + 15x + 8$, where $k \in \mathbb{Z}$, and $x \in \mathbb{R}$.



(a) Given that $f'(3) = -12$, show that $k = -9$, where $f'(3)$ is the derivative of $f(x)$ at $x = 3$.

(b) The function $g(x)$ is the line that passes through the two turning points of $f(x) = x^3 - 9x^2 + 15x + 8$, as shown on the previous page.

Find the equation of $g(x)$.

(c) Show that the graph of $g(x)$ contains the point of inflection of $f(x)$.

<table>
<tr><th>Q4</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$f'(x) = 3x^2 + 2kx + 15$
$3(3)^2 + 2k(3) + 15 = -12$
$27 + 6k + 15 = -12$
$6k = -54$
$k = -9$</td><td>Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
Any relevant differentiation

Mid Partial Credit:
Expression fully differentiated

High Partial Credit:
Derivative fully substituted

No Credit:
No differentiation</td></tr>
<tr><td>(b)</td><td>$f'(x) = 3x^2 + 2(-9)x + 15$
$3x^2 - 18x + 15 = 0$
$x^2 - 6x + 5 = 0$
$x = 1 \quad x = 5$

$f(1) = 15 \qquad (1, 15)$
$f(5) = -17 \qquad (5, -17)$

$m_{g(x)} = -\frac{32}{4} = -8$
$y - 15 = -8(x - 1)$

$g(x)$: $8x + y - 23 = 0$</td><td>Scale 5D (0, 2, 3, 4, 5)
Low Partial Credit:
Any relevant differentiation

Mid Partial Credit:
Both x values found

High Partial Credit:
Turning points found</td></tr>
<tr><td>(c)</td><td>$f''(x) = 6x - 18 = 0$
$x = 3$

$f(3) = -1$

$(3, -1)$ is the point of inflection

$8(3) + (-1) - 23 = 0$
$0 = 0$
$\Rightarrow (3, -1) \in g(x)$.</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$f''(x)$

High Partial Credit:
x coordinate of point of inflection found
Point of inflection found

Note: Accept candidates $g(x)$ from (b) with relevant statement</td></tr>
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