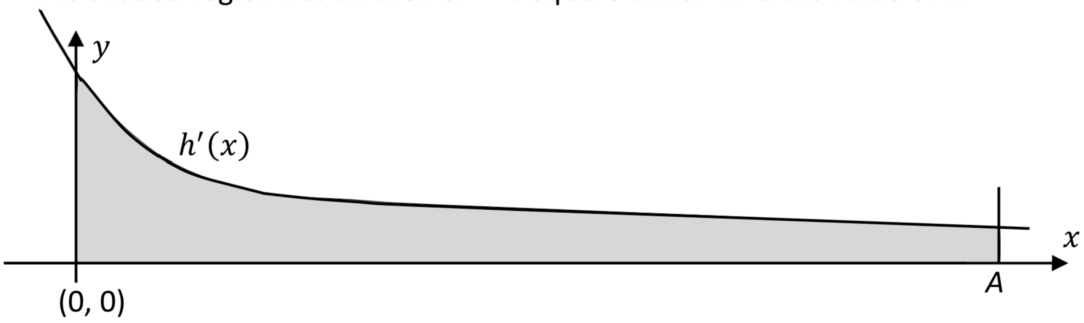
Question 6 **(25 marks)**

(a) Differentiate $(3x-5)(2x+4)$ with respect to x from first principles.

(b) **(i)** $h(x) = \frac{1}{2}\ln(2x+3) + C$, where C is a constant.

Find $h'(x)$, the derivative of $h(x)$.

(ii) The diagram below shows part of the graph of the function $h'(x)$.
The shaded region in the diagram is between the graph and the x-axis, from $x = 0$ to $x = A$.
This shaded region has an area of $\mathbf{ln3}$ square units. Find the value of A.



Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x) = (3x-5)(2x+4)$ $= 6x^2 + 2x - 20$ $f(x+h) = 6(x+h)^2 + 2(x+h) - 20$ $= 6x^2 + 12hx + 6h^2 + 2x + 2h - 20$ $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ $\frac{f(x+h)-f(x)}{h} = 12x + 6h + 2$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = 12x + 2$ $f'(x) = 12x + 2$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into $f(x+h)$ or $y + \Delta y$ *High Partial Credit:* $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ *No Credit*: Not from first principles $(3x-5)(2x+4) = 6x^2 + 2x - 20$
(b) (i)	$h'(x) = \frac{1}{2}\left(\frac{1}{2x+3}\right)(2)$ $= \frac{1}{2x+3}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *High Partial Credit:* $\frac{1}{2}\left(\frac{1}{2x+3}\right)$
(b) (ii)	$\int_0^A \frac{1}{2x+3}\,dx = ln3$ $\frac{1}{2}\ln(2x+3) \;\Big\vert_0^A = \ln 3$ $\frac{1}{2}(\ln(2A+3) - \ln 3) = \ln 3$ $\frac{1}{2}\ln\left(\frac{2A+3}{3}\right) = \ln 3$ $\ln\left(\frac{2A+3}{3}\right)^{\frac{1}{2}} = \ln 3$ $\left(\frac{2A+3}{3}\right)^{\frac{1}{2}} = 3$ $\frac{2A+3}{3} = 9$ $2A + 3 = 27$ $2A = 24$ $A = 12$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* $\frac{1}{2}\ln(2x+3) \;\Big\vert_0^A$ Substitutes limits into integral and stops Correct integration with some substitution *High Partial Credit:* Integral evaluated at $x = A$ only (i.e. omits $\ln 3$ on LHS and finishes **Note**: Must have integration to gain any credit