Question 9 **(55 marks)**

The number of bacteria in the early stages of a growing colony of bacteria can be approximated using the function:

$$N(t) = 450e^{0\cdot065t}$$

where t is the time, measured in hours, since the colony started to grow, and $N(t)$ is the number of bacteria in the colony at time t.

(a) **(i)** Find the number of bacteria in the colony after 4·5 hours.
Give your answer correct to the nearest whole number.

(ii) Find the time, in **hours**, that it takes the colony to grow to 790 bacteria.
Give your answer correct to 1 decimal place.

(b) Using the function $N(t) = 450e^{0\cdot065t}$, find the average number of bacteria in the colony during the period from $t = 3$ to $t = 12$.
Give your answer correct to the nearest whole number.

(c) Find the rate at which $N(t) = 450e^{0\cdot065t}$ is changing when $t = 12$.
Give your answer correct to one decimal place.
Interpret this value in the context of the question.

(d) After k hours, the rate of increase of $N(t)$ is greater than 90 bacteria per hour.
Find the least value of k, where $k \in \mathbb{N}$.

(e) The number of bacteria in the early stages of a **different** colony of bacteria can be approximated using the function:

$$P(t) = 220e^{0\cdot17t}$$

where $P(t)$ is the number of bacteria and t is measured in hours.
Assume that both colonies start growing at the same time.
Find the time, to the nearest hour, at which the number of bacteria in both colonies will be equal.

Q9	Model Solution – 55 Marks	Marking Notes
(a) (i)	$N(t) = 450e^{0\cdot065t}$ $N(4\cdot5) = 450e^{0\cdot065(4\cdot5)}$ $= 602\cdot89$ $= 603$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Correct answer without work *High Partial Credit:* $450e^{0\cdot065(4\cdot5)}$
(a) (ii)	$N(t) = 450e^{0\cdot065t}$ $\frac{N(t)}{450} = e^{0\cdot065t}$ Convert to log equation $\ln(\frac{N(t)}{450}) = 0\cdot065t$ $\ln\left(N(t)\right) - \ln 450 = 0\cdot065t$ $\frac{\ln\left(N(t)\right) - \ln 450}{0\cdot065} = t$ $t = \frac{\ln(790) - \ln 450}{0\cdot065}$ $t = 8\cdot7$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Full substitution and stops *High Partial Credit:* Equation in t (i.e. logs handled correctly)

(b)	$\frac{1}{9}\int_{3}^{12} 450e^{0\cdot065t}dt$ $= \frac{450}{9(0\cdot065)}\left[e^{12(\cdot065)} - e^{3(0\cdot065)}\right]$ $= 743\cdot2$ Average no. $= 743$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* Integration correct *High Partial Credit:* Substitutes limits into integral and stops **Note**: Must have integration to gain any credit
(c)	$N'(t) = 450e^{0\cdot065t} \times 0\cdot065$ $N'(t) = 29\cdot25e^{0.065t}$ $N'(12) = 29\cdot25e^{0\cdot065(12)}$ $= 63\cdot8$ At hour 12 the population is growing at a rate of 64 bacteria per hour **or** At hour 12 the population is growing at a rate of 63·8 bacteria per hour	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $N'(t)$ stated or indicated *High Partial Credit:* Derivative fully substituted $N'(12) = 63\cdot8$ and stops

(d)	$N'(t) = 29{\cdot}25e^{0{\cdot}065k} > 90$ $e^{0{\cdot}065k} > 29{\cdot}25$ $k > \dfrac{\ln \frac{90}{29{\cdot}25}}{0{\cdot}065}$ $k > 17{\cdot}29$ $k = 18$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $29{\cdot}25e^{0{\cdot}065k} > 90$ *High Partial Credit:* Equation in k (i.e. taking logs handled correctly) *No Credit:* No differentiation **Note:** if $k > 17{\cdot}29 \Rightarrow k = 17$ Award Full credit (–1)
(e)	$450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ $\dfrac{450}{220} = \dfrac{e^{0{\cdot}17t}}{e^{0{\cdot}065t}}$ $\dfrac{450}{220} = e^{0{\cdot}105t}$ $\ln\left(\dfrac{450}{220}\right) = 0{\cdot}105t$ $\dfrac{\ln\left(\frac{450}{220}\right)}{0{\cdot}105} = t$ $t = 6{\cdot}82$ $t = 7$ hours	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ *High Partial Credit:* Equation in t (i.e. taking logs handled correctly)