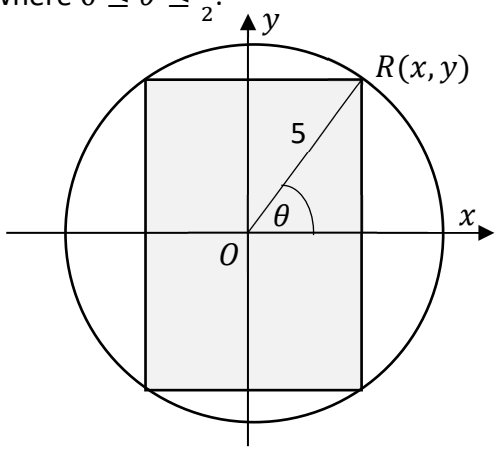
Question 8 **(45 marks)**

A rectangle is inscribed in a circle of radius 5 units and centre $O(0, 0)$ as shown below.
Let $R(x, y)$, where $x, y \in \mathbb{R}$, be the vertex of the rectangle in the first quadrant as shown.
Let θ be the angle between $[OR]$ and the positive x-axis, where $0 \leq \theta \leq \frac{\pi}{2}$.

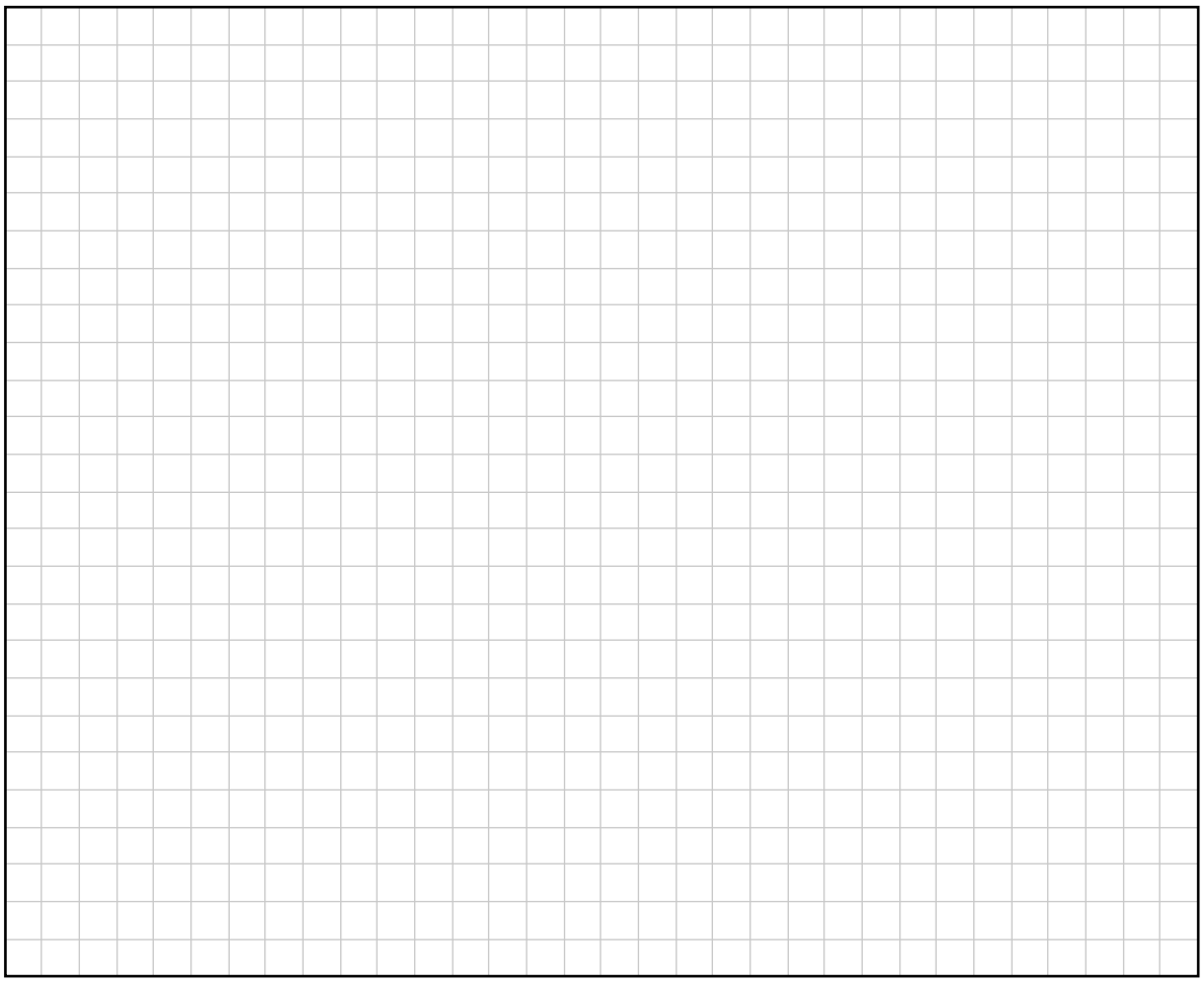


(a) **(i)** The point $R(x, y)$ can be written as $(a \cos \theta, b \sin \theta)$, where $a, b \in \mathbb{R}$.
Find the value of a and the value of b.

$a =$ $b =$

(ii) Show that $A(\theta)$, the area of the rectangle, measured in square units, can be written as $A(\theta) = 50 \sin 2\theta$.

(iii) Use calculus to show that the rectangle with maximum area is a square.



(iv) Find this maximum area.

This question continues on the next page.

(b) A person who is 2 m tall is walking towards a streetlight of height 5 m at a speed of 1·5 m/s. Find the rate, in m/s, at which the length of the person's shadow (x), cast by the streetlight, is changing.