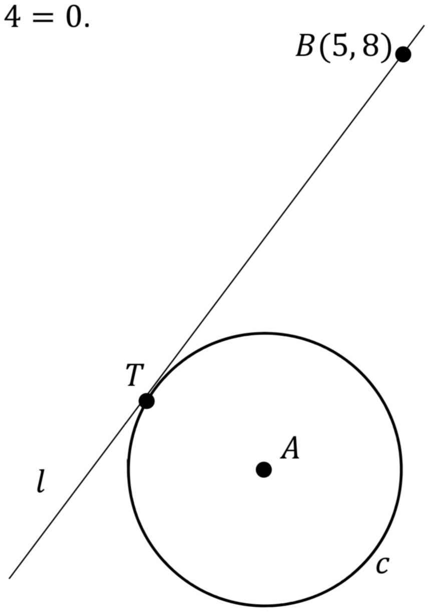
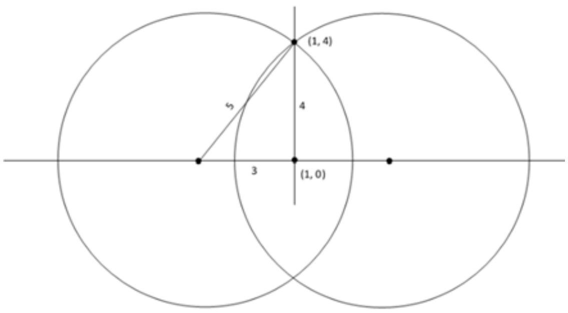
Question 2 **(25 marks)**

(a) The circle c has equation $x^2 + y^2 - 4x + 2y - 4 = 0$.
The point A is the centre of the circle.
The line l is a tangent to c at the point T,
as shown in the diagram.
The point $B(5, 8)$ is on l.
Find $|BT|$.



(b) Two circles, c_1 and c_2, have their centres on the x-axis. Each circle has a radius of 5 units.
The point $(1, 4)$ lies on each circle. Find the equation of c_1 and the equation of c_2.

Q2	Model Solution – 25 Marks	Marking Notes
(a)	Centre: $(2, -1)$ Radius: $\sqrt{2^2 + (-1)^2 + 4} = 3$ Distance from centre to B: $\sqrt{90}$ Pythagoras: $\lvert BT\rvert^2 = 90 - 3^2 = 81$ $\Rightarrow \lvert BT\rvert = 9$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Centre or radius *Mid Partial Credit:* $\sqrt{90}$ *High Partial Credit:* Pythagoras fully substituted (: $\lvert BT\rvert^2$)
(b)	Centre $(-g, 0)$. Radius = $\sqrt{g^2 + (0)^2 - c} = 5$ $\Rightarrow\ g^2 - c = 25$ Equation (i) Equation is $x^2 + y^2 + 2gx + c = 0$ Sub (1, 4): $1^2 + 4^2 + 2g(1) + c = 0$ $\Rightarrow 17 + 2g + c = 0$ Equation (ii) Solve (i) and (ii) $17 + 2g + (g^2 - 25) = 0$ $\Rightarrow\ g^2 + 2g - 8 = 0$ Solve for g: $g = 2$ and $g = -4$ Centres are $(-2, 0)$ and $(4, 0)$ Equations: $(x+2)^2 + y^2 = 25$, $(x-4)^2 + y^2 = 25$ **Or**	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Centre $(-g, 0)$ or equivalent Some substitution of (1, 4) into general equation of circle *Mid Partial Credit:* 2 relevant equations in g and c *High Partial Credit:* Quadratic in g ($g^2 + 2g - 8 = 0$ or equivalent)

	Centre: $(-g, 0)$ $\sqrt{(1+g)^2 + (4-0)^2} = 5$ $(1+g)^2 = 9$ $1 + g = \pm 3$ $g = -4$ or $g = 2$ Equations: $(x+2)^2 + y^2 = 25,$ $(x-4)^2 + y^2 = 25$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Centre $(-g, 0)$ or equivalent Some substitution into distance formula *Mid Partial Credit:* Distance formula fully substituted *High Partial Credit:* Quadratic in g
	or  Centres $(-2, 0)$ and $(4, 0)$; radius =5 Equations: $(x+2)^2 + y^2 = 25,$ $(x-4)^2 + y^2 = 25$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Diagram with (1, 0) identified *Mid Partial Credit:* -2 or 4 identified *High Partial Credit:* $g = -4$ and $g = 2$