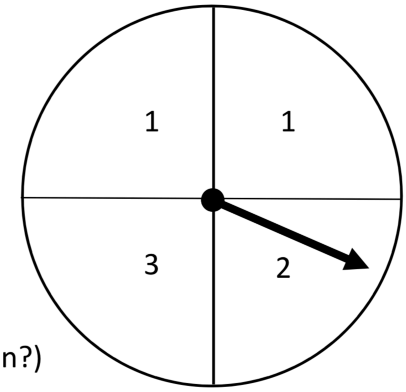
Question 5 **(25 marks)**

(a) Two events A and B are such that $P(A) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{2}$.

(i) Find $P(B|A)$. Give your answer as a fraction in its simplest form.

(ii) $P(A \cup B) = \frac{11}{12}$. Investigate if the events A and B are independent.

(b) A spinner consists of 4 segments, as shown.
Each segment is equally likely to be landed on.
Liam, Sorcha and Lee play a game in which the spinner is spun twice and the numbers landed on are added together.
The result is divided by 3 and the remainder is recorded.



If the remainder is 0 then Liam wins the game.
If the remainder is 1 then Sorcha wins the game.
If the remainder is 2 then Lee wins the game.

Is this a fair game? (i.e. Are all 3 participants equally likely to win?)
Justify your answer by relevant calculations.

Q5	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$$P(B\|A) = \frac{P(A \cap B)}{P(A)}$$ $$P(B\|A) = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula for $P(B\|A)$ *High Partial Credit:* Formula fully substituted
(a) **(ii)**	$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ $$\frac{11}{12} = \frac{3}{4} + P(B) - \frac{1}{2}$$ $$\frac{11}{12} - \frac{1}{4} = P(B) = \frac{2}{3}$$ Check if: $P(A) \times P(B) = P(A \cap B)$ $$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2} = P(A \cap B)$$ $\Rightarrow$ Independent **or** $$P(B\|A) = P(B)$$ $$\frac{2}{3} = \frac{2}{3}$$ $\Rightarrow$ Independent	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Condition for independent events *High Partial Credit:* $P(B) = \frac{2}{3}$ $P(A) \times P(B) = P(A \cap B)$ fully checked for any relevant value (< 1) of $P(B)$ with a valid conclusion

(b)	(tables and conclusion below)	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any relevant listing of remainders/sums *High Partial Credit:* All remainders listed but no conclusion or incorrect conclusion or unsound conclusion

Add	**1**	**1**	**2**	**3**
1	2	2	3	4
1	2	2	3	4
2	3	3	4	5
3	4	4	5	6

Rem.	**1**	**1**	**2**	**3**
1	2	2	0	1
1	2	2	0	1
2	0	0	1	2
3	1	1	2	0

Lee has 6 chances to win.
The others only have 5 chances
$\Longrightarrow$ It is not a fair game