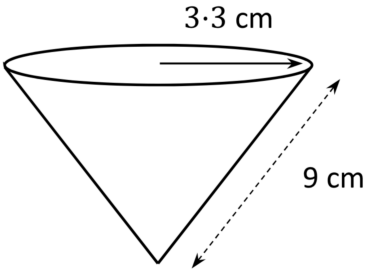
Question 7 **(55 marks)**

(a) A company makes biodegradable paper cups in the shape of a right circular cone. Each cup has a radius of 3·3 cm and a slant height of 9 cm, as shown.

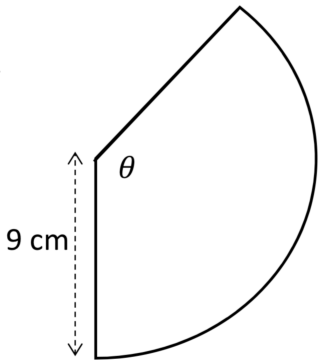


(i) Show that the vertical height of the cup is 8·37 cm, correct to 2 decimal places.

(ii) Find the curved surface area of the cup correct to 2 decimal places.

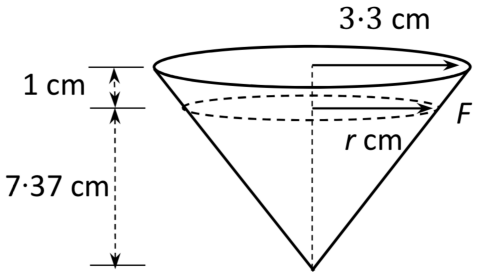
(iii) The diagram shows the net of the cup.

Find, in degrees, the size of the angle θ.

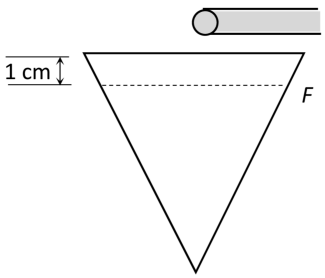


(b) In order to avoid spillages each cup is marked with a dotted line at *F* which is 1 cm vertically below the top of the cup, as shown.

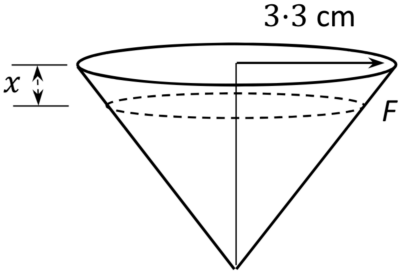
Find the volume of water in the cup when it is filled as far as the dotted line.
Give your answer correct to 1 decimal place.



(c) Water flows into one of these cups through a cylindrical pipe of radius 0·8 cm at a flow rate of 2·5 cm/sec. Find, to the nearest second, how long it will take to fill the cup to the line at *F*.



(d) The company decides to change the position of the line *F* in order to limit the capacity of the cup to 60 cm^3.
How far, vertically below the rim of the cup, should the line *F* be drawn?
Give your answer, in cm, correct to 1 decimal place.



Q7	Model Solution – 55 Marks	Marking Notes
(a) (i)	$9^2 = 3{\cdot}3^2 + h^2$ $h^2 = 81 - 10{\cdot}89$ $h = 8{\cdot}37$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Pythagoras formulated *High Partial Credit:* $\sqrt{9^2 - 3{\cdot}3^2}$ or equivalent
(a) (ii)	$CSA = \pi rl = \pi 3{\cdot}3(9) = 93{\cdot}31$ cm^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formula for **CSA** with some substitution *High Partial Credit:* Formula fully substituted
(a) (iii)	Circumference of cup $= 2\pi r = 2\pi(3{\cdot}3)$ Arc length of sector $= \frac{2\pi \times 9\theta}{360°}$ $2\pi(3{\cdot}3) = \frac{2\pi \times 9\theta}{360°}$ $\theta = \frac{3{\cdot}3(360)}{9} = 132°$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formula for circumference **or** arc length with some substitution *High Partial Credit:* Both formulas fully substituted
(b)	$\frac{3{\cdot}3}{8{\cdot}37} = \frac{r}{7{\cdot}37}$ $r = 2{\cdot}905$ cm $v = \frac{1}{3}\pi(2{\cdot}905)^2 7{\cdot}37$ $65{\cdot}16$ cm^3 $65{\cdot}2$ cm^3	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant effort to find r using similar triangles *Mid Partial Credit:* $\boldsymbol{r}$ found *High Partial Credit:* Volume formula fully substituted Note: If $r = 3{\cdot}3$ used then award MPC at most

(c)	Volume of water in one second $\pi 0{\cdot}8^2(2{\cdot}5)$ $= 5{\cdot}0265\ \text{cm}^3$ Time taken is $\frac{65{\cdot}2}{\pi 0{\cdot}8^2(2{\cdot}5)} = 13$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Any relevant effort to find volume of water *Mid Partial Credit:* $\pi 0{\cdot}8^2(2{\cdot}5)$ *High Partial Credit:* Time formula fully substituted **Note**: Accept work using candidates volume from part (b)
(d)	$\frac{3{\cdot}3}{8{\cdot}37} = \frac{r}{h}$ $r = \frac{3{\cdot}3h}{8{\cdot}37}$ $v = \frac{1}{3}\pi\left(\frac{3{\cdot}3h}{8{\cdot}37}\right)^2 h = 60$ $h^3 = \frac{60 \times 8{\cdot}37^2 \times 3}{\pi 3{\cdot}3^2}$ $h = \sqrt[3]{\frac{60 \times 8{\cdot}37^2 \times 3}{\pi 3{\cdot}3^2}} = 7{\cdot}169$ $x = 8{\cdot}37 - 7{\cdot}169 = 1{\cdot}2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Effort to link r and h *Mid Partial Credit* r and h linked *High Partial Credit:* $h^3 = \frac{60\times 8{\cdot}37^2\times 3}{\pi 3{\cdot}3^2}$ or equivalent