Question 8 **(70 marks)**

(a) An airline company *Trans-sky Airways* has designed an aptitude test for people applying for jobs as trainee pilots. The aptitude test is scored out of 500 marks. The results are normally distributed with a mean score of 280 and a standard deviation of 90.

(i) The top 25% of people taking the aptitude test are invited back for an interview. Find the minimum mark needed on the test in order to be invited back for interview.

(ii) Anyone who scores above the 40^{th} percentile can re-sit the test later.
Eileen scored 260 marks in the test.
Find out whether or not Eileen is eligible to re-sit the test.

(b) **(i)** Explain the relevance of the z-scores $-1{\cdot}96$ and $1{\cdot}96$ in the standard normal distribution.

(ii) *Trans-sky Airways* surveyed 2500 of its passengers about a new service it proposed to introduce. The variable $\hat{p}$ is the proportion of respondents in the survey who said they would use the new service.
The radius of the 95% confidence interval of the survey was 0·01568.
Find the value of $\hat{p}$, where $0{\cdot}5 < \hat{p} \leq 1$.

(c) The weight of the Airline passengers' carry-on luggage is normally distributed with a mean of 12 kg. The Airline has recently introduced a fee for non-carry-on luggage. After the fee was introduced, the Airline expected the mean weight of the carry-on luggage to change.
They selected a random sample of 80 passengers and weighed their carry-on luggage.
The sample mean was 13·1 kg and the sample standard deviation was 4·5 kg.

Test the hypothesis, at the 5% level of significance, that the mean weight of the carry-on luggage has changed. State the null hypothesis and the alternative hypothesis.
Give your conclusion in the context of the question.

(d) The company bus can carry passengers up to a total maximum weight allowance of 3000 kg.
The weight of passengers is normally distributed with a mean of 73 kg and a standard deviation of 12 kg.
40 passengers board the bus.
Find the probability that the total passenger weight will be over the maximum weight allowance.
Give your answer as a percentage correct to 2 decimal places.

(e) A list consists of eight whole numbers. They are labelled from A to H as shown below.
The numbers are all greater than zero and are ordered from smallest to largest.
The difference between any two adjacent numbers is 2 or more.
The median of the list is 12·5.
The lower quartile (the median of the 4 lowest numbers) of the list is 7·5.
The interquartile range is 12.
The second largest number is 23, as shown.
The range of the list is 21.
The mean of the list is 13·5.

Find the numbers which satisfy all of the above conditions and write them into the boxes below.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
						23	

Q8	Model Solution – 70 Marks	Marking Notes
(a) (i)	$$z = \frac{x - \bar{x}}{\sigma}$$ $$\frac{x - 280}{90} = 0{\cdot}68$$ $$\Rightarrow x = 341{\cdot}2$$ $$x = 342$$	**Scale 15D(0, 4, 7, 11, 15)** *Low Partial Credit:* μ or σ identified *Mid Partial Credit:* $0{\cdot}68$ *High Partial Credit:* Equation in x fully substituted and stops or continues incorrectly
(a) (ii)	Eileen's z-score $= \frac{260-280}{90} = -0{\cdot}222 = z$ 40% z-score $= -0{\cdot}25$ i.e. z score for 60% $-0{\cdot}222 > -0{\cdot}25$ Eileen is eligible to re-sit the test. or $P(0{\cdot}222) = 0{\cdot}5871$ $1 - 0{\cdot}5871 = 0{\cdot}4129$ 41·29%	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* μ or σ identified *Mid Partial Credit:* $\frac{260-280}{90}$ or $-0{\cdot}222$ or $-0{\cdot}25$ *High Partial Credit:* $-0{\cdot}222$ and $-0{\cdot}25$ Note: Allow $-0{\cdot}26$
(b) (i)	95% of the of the data lies in the interval $-1{\cdot}96 \leq z \leq 1{\cdot}96$	**Scale 5B (0, 2, 5)** *Partial Credit:* 95% without context

(b) (ii)	$$1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{2500}} = 0{\cdot}01568$$ $$=> \hat{p}(1-\hat{p}) = 2500\left(\frac{0{\cdot}01568^2}{1{\cdot}96^2}\right)$$ $$\Rightarrow \hat{p}^2 - \hat{p} + \frac{4}{25} = 0$$ $$\hat{p} = \frac{1 \pm \sqrt{1-4\left(\frac{4}{25}\right)}}{2} = \frac{1 \pm \frac{3}{5}}{2}$$ $$\hat{p} = \frac{4}{5} \text{ or } \frac{1}{5}$$ $\frac{1}{5}$ outside the range $$\Rightarrow \hat{p} = \frac{4}{5}$$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* $\sqrt{\frac{\hat{p}(1-\hat{p})}{2500}}$ or equivalent written *Mid Partial Credit:* Formula fully substituted *High Partial Credit:* Quadratic in form $a\hat{p}^2 + b\hat{p} + c = 0$
(c)	H_0: Mean weight of bags has not changed H_1: Mean weight of bags has changed $$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{13{\cdot}1 - 12}{\frac{4{\cdot}5}{\sqrt{80}}} = 2{\cdot}186$$ $$2{\cdot}186 > 1{\cdot}96$$ Mean weight of the bags has changed	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* CI formulated with some correct substitution $1{\cdot}96$ H_0 or H_1 *High Partial Credit:* z score fully substituted

(d)		
	$P(\text{weight} > 3000)$ $= P(\text{Average of those on bus} > \frac{3000}{40})$ $P(\bar{x} > 75) = 1 - P(\bar{x} < 75)$ $z = \frac{75-73}{\frac{12}{\sqrt{40}}}$ $= 1{\cdot}054$ This gives a proportion of $0{\cdot}8531$. $1 - 0{\cdot}8531 = 0{\cdot}1469$ $= 14{\cdot}69\%$ This is the probability that the bus with 40 passengers will be above the maximum weight allowance.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{3000}{40}$ μ or σ identified *Mid Partial Credit:* z formula fully substituted *High Partial Credit:* $1{\cdot}054$

(e)	Median is $12 \cdot 5 \Longrightarrow D + E = 25$ LQ is $7 \cdot 5 \Longrightarrow B + C = 15$ IQR is 12 and $12 + 7 \cdot 5 = 19 \cdot 5$ $\Longrightarrow$The upper quartile = $19 \cdot 5$ $F + G = 39$ $G = 23$ so $F = 39 - 23 = 16$ Now $B + C + D + E + F + G = 79$ The total is $8 \times 13 \cdot 5 = 108$ So $A + H = 108 - 79 = 29$ $H - A = 21$ (range) $A = 4$ and $H = 25$ $D + E = 25$ so $D = 11$, $E = 14$ (cannot be 12 and 13 also cannot be 10 and 15) $B + C = 15$ so $B = 6$, $C = 9$ (cannot be 7 and 8 also cannot be 5 and 10) The list is: A B C D E F G H 4 6 9 11 14 16 23 25	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* One unknown number given One relevant equation written *Mid Partial Credit* Three unknown numbers given Three relevant equations written *High Partial Credit:* Five unknown numbers given Five relevant equations written