Leaving Certificate 2022

Mathematics

Higher Level

Paper 2

Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D	E
No of categories	3	4	5	6
5-mark scale	0, 2, 5	0, 2, 3, 5		
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10	
15-mark scale			0, 4, 8, 12, 15	0, 3, 6, 9, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

etc.

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. A penalty for an omitted unit is applied only once in each section (a), (b), (c) etc. There is no penalty for omitted units if the question specifies the unit to be used in the answer.

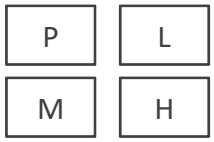
In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Unless otherwise specified, a correct answer without sufficient supporting work is generally awarded the highest level of partial credit (typically *Partial Credit* or *High Partial Credit*, as appropriate).

Summary of mark allocations and scales to be applied

Section A (120) **Answer any four questions**		**Section B (100)** **Answer any two questions**	
Question 1 (30)	**Question 4 (30)**	**Question 7 (50)**	**Question 9 (50)**
(a)(i) 5C	(a)(i) 10C	(a) 10C	(a) 10D
(a)(ii) 10D	(a)(ii) 10C	(b) 10D	(b) 10C
(b) 5C	(b) 10C	(c) 10D	(c)(i) 5C
(c) 10D		(d)(i) 5B	(c)(ii) 10C
	Question 5 (30)	(d)(ii) 10D	(d)(i) 5B
Question 2 (30)	(a)(i)(ii) 10C	(e) 5C	(d)(ii) 10D
(a) 10C	(a)(iii) 5C		
(b) 5C	(b) 15D	**Question 8 (50)**	**Question 10 (50)**
(c) 15D		(a)(i)(ii) 10C	(a)(i) 10D
	Question 6 (30)	(a)(iii)(iv) 5C	(a)(ii) 5C
Question 3 (30)	(a) 15D	(a)(v) (b) 5B	(b) 5C
(a) 5C	(b) 10D	(c) 10D	(c) 10D
(b) 10C	(c) 5B	(d)(i) 10C	(d) 5C
(c) 15D		(d)(ii) 10C	(e) 15E

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
	Tick	Work of relevance	The work presented in the body of the script merits full credit
	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
			The work presented in the body of the script merits the relevant level of partial credit (*Partial*, *Low Partial*, *Mid Partial*, and *High Partial* respectively)
	F star		The work presented in the body of the script merits *Full Credit – 1*
	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
	Oversimplify	The candidate has oversimplified the work	
	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and  and  appear in the body of the work, then  should be placed in the right margin.

In the case of a **D scale** with the same annotations,  should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. It is specifically noted where this does not hold. Note also that these steps may not need to be presented in the order specified in the Marking Notes.

Where "finishes correctly" is included in the Marking Notes, this is taken to mean: "finishes using the correct method, and the (incorrect) values the candidate has already found".

<table>
<tr><th>Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>
<table>
<tr><td></td><td colspan="2">Age (years)</td><td rowspan="2">Total</td></tr>
<tr><td></td><td>$\leq$ 23</td><td>$\geq$ 24</td></tr>
<tr><td>Under.</td><td>12 785</td><td>2922</td><td>15 707</td></tr>
<tr><td>Post.</td><td>1353</td><td>5654</td><td>7007</td></tr>
<tr><td>Total</td><td>14 138</td><td>8576</td><td>22 714</td></tr>
</table>
</td><td>Scale 5C (0, 2, 3, 5)

Accept correct answers without supporting work

Low Partial Credit
• 1 value correct

High Partial Credit
• 2 values correct</td></tr>
</table>

Q1	Model Solution – 30 Marks	Marking Notes
(a) (ii)	**Conclusion**: They are **not** independent **Justification**: *Based on independent iff* $P(O).P(U) = P(O \cap U)$: $P(O) = \frac{8576}{22714}$ or $\frac{4288}{11357}$ or $0{\cdot}377..$ $P(U) = \frac{15707}{22714} = 0{\cdot}6915..$ $P(O) \times P(U) = 0{\cdot}261\ ...$ $P(O \cap U) = \frac{2922}{22714} = 0{\cdot}1286 \neq P(O).P(U)$ **OR** *Based on independent iff* $P(O) = P(O\|U)$: $P(O) = \frac{8576}{22714}$ or $\frac{4288}{11357}$ or $0{\cdot}377..$ $P(O\|U) = \frac{2922}{15707} = 0 \cdot 186..$ **OR** *Based on independent iff* $P(U) = P(U\|O)$: $P(U) = \frac{15707}{22714} = 0{\cdot}691$ $P(U\|O) = \frac{2922}{8576} = 0{\cdot}3407$ **OR** Relatively few of the undergraduates are 24 or older, compared to the university overall. **OR** Over half the students in the university are undergraduates, but only about one third of students aged 24 or older are undergrads	**Scale 10D (0,3,5,8,10)** Accept without "$\neq \boldsymbol{P(O).P(U)}$" or similar, if conclusion is correct Note: calculations that do not involve probabilities are not awarded credit *Low Partial Credit* • Work of merit, for example, correct conclusion, or makes a relevant statement, or finds a relevant probability *Mid Partial Credit* • Finds two of $P(O)$, $P(U)$, $P(O) \times P(U)$, or $P(O \cap U)$ • Finds $P(O\|U)$ or $P(U\|O)$ *High Partial Credit* • Enough calculations to support a correct conclusion, but no conclusion • Correct conclusion **and** finds $P(O) \times P(U)$ • Correct conclusion **and** finds $P(O)$, $P(U)$, and $P(O \cap U)$ • Correct conclusion **and** finds $P(O\|U)$ or $P(U\|O)$
(b)	$\frac{1}{7} \times \frac{1}{7}$ or $\frac{7}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$ or $7 \times \left(\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}\right) = \frac{1}{49}$	**Scale 5C (0,2,3,5)** Accept correct answer without supporting work *Low Partial Credit* • Work of merit, for example, one correct probability *High Partial Credit* • $\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$ or $\frac{1}{343}$

Q1	Model Solution – 30 Marks	Marking Notes
(c)	$\frac{g}{b+g}=\frac{3}{5} \Rightarrow 3b-2g=0$ or $b=\frac{2}{3}g$ $\frac{g+4}{b+g+8}=\frac{4}{7} \Rightarrow -4b+3g=4$ $9b-6g=0$ $-8b+6g=8$ $b=8$ $3b=2g \Rightarrow 2g=24 \Rightarrow g=12$ **OR** x students in class : $\frac{g}{x}=\frac{3}{5}$ so $g=\frac{3}{5}x$ $x+8$ in class: $\frac{g+4}{x+8}=\frac{4}{7}$ so $7g+28=4x+32$ Sub 1st into 2nd: $7\left(\frac{3}{5}x\right)+28=4x+32$ Solve: $x=20$, so $g=12$ and $b=8$ **OR** *Trial and Improvement*: 2 boys and 3 girls becomes 6 boys and 7 girls, so $P=\frac{7}{13}$ (not correct) 4,6 becomes 8, 10, so $P=\frac{10}{18}$ (not correct) 8,12 becomes 12, 16, so $P=\frac{16}{28}=\frac{4}{7}$ So $b=8$, $g=12$	**Scale 10D (0,3,5,8,10)** **1.** Initial equation in b and g **2.** Subsequent equation in b and g **3.** Finds b **4.** Finds g Accept verified correct answer (gives b and g, finds $b+4$ and $g+4$, and verifies that probability is correct) *Low Partial Credit* • Work of merit, for example, indicates $\frac{2}{5}$, or indicates $b+g$, or some part of fraction involving b and g correct, or indicates use of additional 4 boys or girls or total of 8 in a relevant probability *Mid Partial Credit* • 1 step correct • 2 or more incorrect possibilities for b and g tested (by adding 4 to each and finding resulting probability) *High Partial Credit* • 2 steps correct • 2 correct linear equations in 2 unknowns • Correct answer without supporting work or verification *Full Credit –1* • Finds one variable (b or g)

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$C = \left(\frac{1(8)+4(-1)}{4+1}, \frac{1(-4)+4(3)}{4+1}\right)$ $C = \left(\frac{4}{5}, \frac{8}{5}\right)$ **OR** x: 9 steps back $[8$ to $-1]$ So $x_C = 8 - \frac{4}{5}(9) = \frac{4}{5}$ y: 7 steps up $[-4$ to $3]$ So $y_C = -4 + \frac{4}{5}(7) = \frac{8}{5}$ $C = \left(\frac{4}{5}, \frac{8}{5}\right)$	**Scale 10C (0,3,7,10)** *Low Partial Credit* • Correct formula with some substitution • Value from formula explicitly identified $(a, b, x_1, y_1, x_2, y_2)$ • Recognises 9 steps for x or 7 steps for y • Plots both points on a set of axes *High Partial Credit* • Fully correct substitution into formula • $8 + \frac{4}{5}(-1-8)$ **and** $-4 + \frac{4}{5}\big(3-(-4)\big)$, or equivalent
(b)	From y-intercept to (q, r): Run $= q$, so rise $= qm$, so y-value $= r - qm$ Answer: $(0, r - qm)$ **OR** $y = mx + c$ So $r = mq + c$ So $c = r - mq$ Answer: $(0, -mq + r)$ **OR** $y - y_1 = m(x - x_1)$ $y - r = m(x - q)$ $y - r = mx - mq$ $y = mx - mq + r$ $x = 0$, so $y = -mq + r$ Answer: $(0, -mq + r)$	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, equation of line formula (accept m as substitution), or indicates distance q or r correctly on diagram, or indicates $x = 0$ *High Partial Credit* • States $x = 0$ **and** work of merit towards finding y • Correct y-value found *Full Credit –1* • Correct values for x and y, but not given as co-ordinates of a point

Q2	Model Solution – 30 Marks	Marking Notes
(c)	$\tan 30° = \frac{-2-m_2}{1+(-2)m_2}$ So $\frac{1}{\sqrt{3}} = \frac{-2-m_2}{1-2m_2}$ $1 - 2m_2 = -2\sqrt{3} - \sqrt{3}m_2$ $m_2 = \frac{1+2\sqrt{3}}{2-\sqrt{3}}$ $m_2 = \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $m_2 = 8 + 5\sqrt{3}$ **OR** $\frac{1}{\sqrt{3}} = \frac{-(-2-m_2)}{1+(-2)m_2}$ $1 - 2m_2 = 2\sqrt{3} + \sqrt{3}m_2$ $m_2 = \frac{1-2\sqrt{3}}{2+\sqrt{3}}$ $m_2 = \frac{1-2\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ $m_2 = 8 - 5\sqrt{3}$	**Scale 15D (0,4,8,12,15)** **1.** Subs in 30 or -2 **2.** Fully substituted formula (30 and -2) **3.** Eliminate fractions and expands **4.** Find slope Note: A solution based on $tan^{-1}(-2) = 116{\cdot}5\ldots°$ is awarded at most *Low Partial Credit* *Low Partial Credit* • Work of merit, for example, formula for $\tan\theta$ with substitution for given slope, or $\tan 30°$ indicated • Indicates $\tan^{-1}(-2)$ or $116{\cdot}5\ldots°$ *Mid Partial Credit* • 2 steps correct • Sets $30 = \frac{-2-m_2}{1-2m_2}$ and finishes correctly *High Partial Credit* • 3 steps correct *Full Credit –1* • Correct value for slope, but not in required form

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Centre $= (1, -4)$ Radius $= \sqrt{(-1)^2 + (4)^2 - k} = 5\sqrt{3}$ $17 - k = 75$ $k = -58$ **OR** $x^2 - 2x + 1 + y^2 + 8y + 16 + k - 17 = 0$ $r^2 = 17 - k = \left(5\sqrt{3}\right)^2$ $k = 17 - \left(5\sqrt{3}\right)^2 = -58$	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, some correct substitution into a relevant formula, or centre identified, or completes square for x or y in the equation of the circle *High Partial Credit* • Finds an equation in k
(b)	Centre $= (5, -2)$ $m_{Normal} = \frac{-4-(-2)}{9-5} = -\frac{1}{2}$ $m_{Tangent} = 2$ **OR** $(x-5)^2 + (y+2)^2 = 20$ $2(x-5) + 2(y+2)\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-x+5}{y+2}$ At $(9, -4)$, slope $= \frac{-9+5}{-4+2} = 2$	**Scale 10C (0,3,7,10)** First solution: **1.** Identifies centre correctly **2.** Finds slope of normal correctly **3.** Finds slope of tangent correctly *Low Partial Credit* • Work of merit, for example, some substitution into slope or line formula, or identifies centre, or some correct differentiation *High Partial Credit* • 2 steps correct • $\frac{dy}{dx} = \frac{-x+5}{y+2}$ • Finds equation of tangent (without explicitly finding the slope)

Q3	Model Solution – 30 Marks	Marking Notes
(c)	Centre $= (r, -r)$ $(1-r)^2 + \left(-8-(-r)\right)^2 = r^2$ $1 - 2r + r^2 + 64 - 16r + r^2 = r^2$ $r^2 - 18r + 65 = 0$ $(r-13)(r-5) = 0$ $r = 13$ or $r = 5$ *Answers*: $(x-13)^2 + (y+13)^2 = 169$ $(x-5)^2 + (y+5)^2 = 25$ **OR** $x^2 + y^2 + 2gx + 2fy + c = 0$ $1^2 + (-8)^2 + 2g(1) + 2f(-8) + c = 0$ $2g - 16f + c = -65$ $f = -g$ and $\lvert g\rvert = r = \sqrt{g^2 + f^2 - c}$ $g^2 = g^2 + g^2 - c \Rightarrow g^2 = c$ $2g - 16(-g) + g^2 = -65$ $g^2 + 18g + 65 = 0$ $(g+13)(g+5) = 0$ $g = -13$ or $g = -5$ $f = 13$ or $f = 5$ $r = 13$ and 5 *Answers*: $x^2 + y^2 - 26x + 26y + 169 = 0$ $x^2 + y^2 - 10x + 10y + 25 = 0$	**Scale 15D (0,4,8,12,15)** *Low Partial Credit* • Work of merit, for example, indicates centre is $(r, -r)$, or some substitution of $(1, -8)$ into equation of circle, or finds $c = g^2$, or $\lvert g\rvert = \lvert f\rvert$ *Mid Partial Credit* • Finds correct quadratic in one variable (r, g, f, or c), fully expanded *High Partial Credit* • Finds both values of r, f, or g • Finds one value of r, f, and g, **and** the equation of the associated circle

Q4	Model Solution – 30 Marks	Marking Notes
(a) (i)	$\tan(A+(-B)) = \frac{\tan A+\tan(-B)}{1-\tan A\tan(-B)}$ $= \frac{\tan A-\tan B}{1+\tan A\tan B}$ **OR** $\frac{\tan A-\tan B}{1+\tan A\tan B} = \frac{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}}{1+\frac{\sin A}{\cos A}\times\frac{\sin B}{\cos B}}$ $= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$ $= \frac{\sin(A-B)}{\cos(A-B)} = \tan(A-B)$ **OR** $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$ $= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$ $= \frac{\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B}+\frac{\sin A\sin B}{\cos A\cos B}}$ $= \frac{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}}{\frac{1}{1}+\frac{\sin A\sin B}{\cos A\cos B}}$ $= \frac{\tan A-\tan B}{1+\tan A\tan B}$	**Scale 10C (0,3,7,10)** *Low Partial Credit* • Work of merit, for example, indicates that $A-B=A+(-B)$, or writes $\tan A = \frac{\sin A}{\cos A}$ *High Partial Credit* • $\frac{\tan A+\tan(-B)}{1-\tan A\tan(-B)}$ • $\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$

Q4	Model Solution – 30 Marks	Marking Notes
(a) (ii)	$\tan 15 = \tan(60 - 45)$ $= \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ $= \frac{\sqrt{3}-1}{\sqrt{3}+1}$ **OR** $\tan 15 = \tan(45 - 30)$ $= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$ $= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ **OR** $\tan 30 = \frac{1}{\sqrt{3}} = \frac{2 \tan 15}{1 - \tan^2 15}$ $1 - \tan^2 15 = 2\sqrt{3} \tan 15$ $\tan^2 15 + 2\sqrt{3} \tan 15 - 1 = 0$ $\tan 15 = \frac{-2\sqrt{3} \pm \sqrt{(-2\sqrt{3})^2 - 4 \times 1 \times (-1)}}{2}$ $= -\sqrt{3} + 2$ (first quadrant) $= \frac{\sqrt{3}-1}{\sqrt{3}+1}$	**Scale 10C (0,3,7,10)** Note: accept solution of: $\tan 15° = -\sqrt{3} + 2 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ *Low Partial Credit* • Work of merit, for example, $60 - 45$, or some correct substitution into relevant formula, or top line of relevant formula substituted • $-\sqrt{3} + 2$ with no supporting work *High Partial Credit* • $\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ or equivalent • Makes error(s) in arriving at $\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ (or equivalent) from relevant expression, but finishes correctly • $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ without supporting work • $\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$ or $-\sqrt{3} + 2$, with supporting work (equivalent to that in model solutions)

Q4	Model Solution – 30 Marks	Marking Notes
(b)	*Sine Rule*: $\frac{180-45}{2} = 67{\cdot}5°$ $\frac{x}{\sin 67{\cdot}5} = \frac{10\sqrt{2-\sqrt{2}}}{\sin 45}$ $x = \frac{10\sqrt{2-\sqrt{2}}\ \sin 67{\cdot}5}{\sin 45} = 10$ **OR** *Cosine Rule:* $\left(10\sqrt{2-\sqrt{2}}\right)^2 = x^2 + x^2 - 2(x)(x)\cos 45°$ $\left(2-\sqrt{2}\right)x^2 = 100\left(2-\sqrt{2}\right)$ $x = 10$ [as $x > 0$] **OR** *Drop a perpendicular from C:* $\cos 67{\cdot}5 = \frac{5\sqrt{2-\sqrt{2}}}{x}$ $x = \frac{5\sqrt{2-\sqrt{2}}}{\cos 67{\cdot}5} = 10$	**Scale 10C (0,3,7,10)** *Low Partial Credit* • Work of merit, for example, some correct substitution into a relevant formula, or finds angle at A or B, or draws perpendicular from C (right angle indicated) *High Partial Credit* • Fully substituted formula (Sine Rule, Cosine Rule, or $\cos 67{\cdot}5$) *Full Credit –1* • Early rounding and finishes correctly

Q5	Model Solution – 30 Marks	Marking Notes
(a) **(i)** **(ii)**	**(i)** $\hat{p} = \frac{135}{400} = 0{\cdot}3375$ **(ii)** $0{\cdot}3375 \pm \frac{1}{\sqrt{400}}$ $= 0{\cdot}3375 \pm 0{\cdot}05$ $= [0{\cdot}2875, 0{\cdot}3875]$	**Scale 10C (0,3,7,10)** In (i), accept correct answer without work Accept answers given as percentages, correct to 2 or 4 decimal places *Low Partial Credit* • Work of merit, for example, indicates $\frac{1}{\sqrt{400}}$ in (ii) *High Partial Credit* • (i) or (ii) correct
(a) **(iii)**	$\hat{p} \pm 1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= 0{\cdot}3375 \pm 1{\cdot}96\sqrt{\frac{0{\cdot}3375(1-0{\cdot}3375)}{400}}$ $= 0{\cdot}3375 \pm 0{\cdot}04633\ldots$ $= [0{\cdot}2912, 0{\cdot}3838]$ [4 D.P.]	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Some substitution into correct formula, or substantial part thereof • Indicates $1{\cdot}96$, or $1 - \hat{p}$ *High Partial Credit* • Fully substituted correct formula • Finds the maximum or minimum of the confidence interval
(b)	*Null Hypothesis*: Average [mean] amount has not changed *Alternative Hypothesis*: Average [mean] amount has changed *Conclusion*: The average amount has changed *Calculations & Reason*: $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{22{\cdot}16-20{\cdot}79}{\frac{8{\cdot}12}{\sqrt{500}}} = 3{\cdot}7726\ldots$, which is greater than $1{\cdot}96$ **OR** $20{\cdot}79 \pm 1{\cdot}96\frac{8{\cdot}12}{\sqrt{500}} = [20{\cdot}07.., 21{\cdot}50..]$, and $22{\cdot}16$ lies outside this **OR** $22{\cdot}16 \pm 1{\cdot}96\frac{8{\cdot}12}{\sqrt{500}} = [21{\cdot}44.., 22{\cdot}87\ldots]$, and $20{\cdot}79$ lies outside this	**Scale 15D (0,4,8,12,15)** **1.** Hypotheses **2.** Calculations (sufficient to support a conclusion) **3.** Conclusion (not considered correct without some relevant calculations) **4.** Reason (must match calculations) *Low Partial Credit* • Work of merit, for example, one hypothesis stated correctly, some relevant calculation indicated, identifies μ or σ, hypotheses swapped *Mid Partial Credit* • Calculations correct • 2 steps correct *High Partial Credit* • Calculations correct **and** one other step

Q6	Model Solution – 30 Marks	Marking Notes
(a)	*Circumcentre correctly drawn, with construction lines, and labelled C*	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit* • Some indication of understanding of perpendicular bisector or midpoint, or some evidence of relevant use of compass • Work towards finding the incentre, orthocentre, or centroid *Mid Partial Credit* • 1 perpendicular bisector constructed (with construction lines) • 2 non-intersecting perpendicular bisectors drawn, without construction lines *High Partial Credit* • 2 non-intersecting perpendicular bisectors constructed (with construction lines) • 2 intersecting perpendicular bisectors drawn, without construction lines • Centre correct, no construction lines *Full Credit –1* • 2 intersecting perpendicular bisectors constructed (with construction lines), but circumcentre not labelled

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$\lvert\angle ADB\rvert = 90°$ [*angle in semi-circle*] So $\lvert\angle ABD\rvert = 45°$ [*isosceles, and angles in a* Δ] So $\lvert\angle ACD\rvert = 45°$ [*standing on the same arc*] So $\lvert\angle ADC\rvert = 180 - 45 - 40 = 95°$ **OR** $\lvert\angle ADB\rvert = 90°$ [*angle in semi-circle*] So $\lvert\angle ABD\rvert = 45°$ [*isosceles, and angles in a* Δ] So $\lvert\angle CDB\rvert = \lvert\angle BAC\rvert = 5°$ So $\lvert\angle ADC\rvert = 90 + 5 = 95°$	**Scale 10D (0,3,5,8,10)** *Low Partial Credit* • Work of merit, for example, indicates $90°$ or right angle or $\lvert AD\rvert = \lvert DB\rvert$ *Mid Partial Credit* • Finds $45°$ angle(s) in triangle ADB • Indicates two equal angles standing on same arc *High Partial Credit* • Indicates two equal angles standing on same arc, **and** indicates $90°$ or uses given isosceles triangle
(c)	*Most of proof may be given as construction* Join P to O to Q. The angle at the centre ($\lvert\angle POQ\rvert$) must be twice $\lvert\angle PRQ\rvert$, so must be $> 180°$. So, the point O has to be outside the triangle PQR. **OR** Assume that O **is** inside the triangle PQR. Then $\lvert\angle POQ\rvert < 180°$, where $\angle POQ$ is the angle in the triangle POQ. But $\lvert\angle PRQ\rvert$ is half this angle, so $\lvert\angle PRQ\rvert < 90°$, a contradiction. **OR** Assume that O **is** inside the triangle PQR. Join P to O and continue until you hit the circle k, on the minor arc between R and Q. Label this point S. Then $\lvert\angle PRS\rvert = 90°$, as $[PS]$ is a diameter. But $\lvert PRS\rvert > \lvert\angle PRQ\rvert$, a contradiction. **OR** The line through R perpendicular to PR intersects k on the arc joining P to Q that doesn't contain R. Label this point T. Join T to P. The centre is the midpoint of $[TP]$, which is outside the triangle PQR.	**Scale 5B (0, 2, 5)** Note: if proof by contradiction is presented, assume that given first line has been used, even if not stated by candidate. *No Credit* • Writes down given first line and no other relevant work • Draws / constructs perpendicular bisector of any / all sides *Partial Credit* • Performs construction / description of construction that would support proof, but proof not given • Relevant statement made, for example, "if R is a right angle, centre is on PQ."

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$V = \pi r^2 h = 450\pi$ $(5^2)\ h = 450$ $h = 18$ cm	**Scale 10C (0,3,7,10)** *Low Partial Credit* • Work of merit, for example, finds radius, or sets formula for volume equal to 450π *High Partial Credit* • Fully substituted correct equation • Uses an incorrect volume formula, but finishes correctly • Mishandles diameter / radius, but finishes correctly *Full Credit –1* • Correct answer but no or incorrect unit
(b)	$V_{\text{small}} = \frac{1}{3}\pi r^2 h = 12\pi$ so $r^2 h = 36$ $V_{\text{large}} = \frac{1}{3}\pi(kr)^2(2h) = 150\pi$ so $k^2 r^2 h = 225$ So $k^2(36) = 225$ $k = \sqrt{\frac{225}{36}} = \frac{5}{2}$ or 2·5	**Scale 10D (0,3,5,8,10)** **1.** Sets up equation based on V_{small} **2.** Sets up equation based on V_{large} **3.** Creates single equation in k^2 **4.** Finds k from equation in k^2 *Low Partial Credit* • Work of merit, for example, some correct substitution into volume formula *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q7	Model Solution – 50 Marks	Marking Notes
(c)	Length of arc from B to A: $= \frac{216}{360} \times 2\pi 8 = 9{\cdot}6\,\pi$ [cm] Let radius of cone $= r$: $2\pi r = 9{\cdot}6\,\pi$ so $r = 4{\cdot}8$ [cm]	**Scale 10D (0,3,5,8,10)** Accept length of major or minor arc. However, length of minor arc cannot be used directly to find radius of cone. *Low Partial Credit* • Work of merit, for example, correct substitution into a relevant formula, or indicates that the arc-length from B to A is the circumference of the cone *Mid Partial Credit* • Length of arc fully substituted *High Partial Credit* • Length of arc correct, and work of merit in the other part • Finds radius of cone based on length of minor arc • Finds radius of cone based on curved surface area, without finding length of arc
(d) (i)	$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2{\cdot}7)^3$ $= 82{\cdot}4479 \ldots = 82{\cdot}448$ cm^3 [3 D.P.]	**Scale 5B (0,2,5)** Accept correct answer without supporting work *Partial Credit* • Fully substituted formula

Q7	Model Solution – 50 Marks	Marking Notes
(d) (ii)	Let $r =$ radius of circular base: $\pi r^2 = 5{\cdot}4 \Rightarrow r = 1{\cdot}311\ldots$ Let $x =$ vertical height of centre of sphere above horizontal cut: $x^2 + (1{\cdot}311\ldots)^2 = 2{\cdot}7^2$ $x = \sqrt{7{\cdot}29 - 1{\cdot}718\ldots} = 2{\cdot}36\ldots$ $l = 2{\cdot}7 + 2{\cdot}36\ldots = 5{\cdot}06 = 5{\cdot}1$ [cm] [1 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** **1.** Sets up equation involving area of circular base **2.** Finds radius of circular base **3.** Sets up quadratic equation in x **4.** Finds x and hence finds l *Low Partial Credit* • Work of merit, for example, marking $2 \cdot 7$ correctly, or showing $l > 2 \cdot 7$ on diagram *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Finds x correctly, but fails to find l
(e)	$\lvert EB\rvert = 15$ $\lvert EC\rvert = \sqrt{15^2 + 30^2} = 15\sqrt{5}$ ΔEOB is similar to ΔEBC, so $\frac{\lvert EO\rvert}{\lvert EB\rvert} = \frac{\lvert EB\rvert}{\lvert EC\rvert}$ $\lvert EO\rvert = \frac{15^2}{15\sqrt{5}} = 3\sqrt{5}$ [cm] **OR** $\lvert\angle EBO\rvert = \lvert\angle BCE\rvert$ and $\tan BCE = \frac{1}{2}$ $\sin EBO = \frac{\lvert EO\rvert}{15}$ and $\sin EBO = \sin BCE = \frac{1}{\sqrt{1^2+2^2}} = \frac{1}{\sqrt{5}}$ So $\lvert EO\rvert = \frac{15}{\sqrt{5}} = 3\sqrt{5}$ [cm]	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, indicates two similar triangles, or two equal angles, or a correct trigonometric ratio of a relevant angle, or 15 *High Partial Credit* • States $\frac{\lvert EO\rvert}{\lvert EB\rvert} = \frac{\lvert EB\rvert}{\lvert EC\rvert}$ **and** has also found $\lvert EB\rvert$ and $\lvert EC\rvert$ • $\tan BCE = \frac{1}{2}$ **and** $\sin EBO = \frac{\lvert EO\rvert}{15}$, or similar *Full Credit –1* • Correct value found, but not in the correct form • Finds $\lvert OB\rvert$ (which is $6\sqrt{5}$ cm)

Q8	Model Solution – 50 Marks	Marking Notes
(a) **(i)** **(ii)**	**(i)** ***G*** *and* ***H*** *plotted and labelled correctly* **(ii)** *Reasonable line of best fit drawn*	**Scale 10C (0,3,7,10)** In **(i)**, accept x_H plotted between 15 and 20 In **(ii)**, accept line of best fit with some values on each side and with reasonable slope *Low Partial Credit* • Work of merit in plotting **G** or **H**, for example, one ordinate correct *High Partial Credit* • (i) or (ii) correct *Full Credit –1* • Points and line correctly plotted, but no or incorrect labels on points
(a) **(iii)** **(iv)**	**(iii)** *Answers consistent with candidate's line of best fit* **(iv)** Answer: **K** Reason: **L** is well beyond all of the given data points	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, in (iii): relevant work on the graph, or in (iv): gives **K**, or reason shows some relevant knowledge *High Partial Credit* • One part correct • Work of merit in both parts *Full Credit –1* • Correct answers and reason, but without supporting work on the graph for (iii)

Q8	Model Solution – 50 Marks	Marking Notes
(a)(v) **(b)**	**(a)(v)** $0{\cdot}9659 \ldots = 0{\cdot}966$ [3 D.P.] **(b)** *Any valid reason, for example:* Correlation is related to linear relationships, and this is quadratic The line of best fit is close to horizontal The data is symmetrical: it is decreasing first, and then increasing in a symmetrical way	**Scale 5B (0,2,5)** In (a)(v), accept correct answer without supporting work *Partial Credit* • Work of merit, for example, in (a)(v), fully substituted correct formula; or, in (b), draws axis of symmetry or line of best fit *Full Credit –1* • (b) correct and $0{\cdot}965$ or $0{\cdot}96$ or $0{\cdot}97$ given in (a)(v)
(c)	Mean $= \frac{534 + S + M}{13} = 52$ $534 + S + M = 676$ $S + M = 142$ Median $= 54$, so the least value S could be is 55 Greatest value of $S = 142 - 55 = 87$	**Scale 10D (0,3,5,8,10)** *Low Partial Credit* • Work of merit, for example, finds total of the scores (including or excluding S and/or M), or work towards finding the median (for example, rewriting the values in ascending order) *Mid Partial Credit* • Correct equation in S and M • Least value $= 55$ *High Partial Credit* • Correct equation in S and M **and** indicates effort to find values greater than 54 • Finds answers of 54 and 88, 56 and 86, etc. (that is, $S + M = 142$ and incorrect values, both ≥ 54, found) • Correct answers (55 and 87) with insufficient supporting work

Q8	Model Solution – 50 Marks	Marking Notes
(d) (i)	**Working out**: Doesn't replace: $E(X) = 0{\cdot}095(20\,000) = 1900$ Replaces: $E(X) = 1450 + 0{\cdot}005(20\,000) = 1550$ **Conclusion**: He should replace it now [as expected cost is less if he replaces it]. Note: candidates may find both expected values at once, with a minus in between, to evaluate which is bigger	**Scale 10C (0,3,7,10)** *No Credit* • Correct conclusion with no supporting work *Low Partial Credit* • Work of merit, for example, indicates one relevant operation *High Partial Credit* • Both expected values found, no or incorrect conclusion • One correct and one incorrect expected value found, with the correct (consistent) conclusion
(d) (ii)	$P(\text{at least } 1) = 31 - P(\text{none})$ $= 1 - 0{\cdot}905 \times 0{\cdot}959 \times 0{\cdot}927$ $= 1 - 0{\cdot}8045 \ldots$ $= 0{\cdot}19547 \ldots = 0{\cdot}195$ [3 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • 1 relevant probability found • 1st line of solution • $0{\cdot}095 \times 0{\cdot}041 \times 0{\cdot}073$ *High Partial Credit* • $0{\cdot}905 \times 0{\cdot}959 \times 0{\cdot}927$

Q9	Model Solution – 50 Marks	Marking Notes
(a)	Area Field $1 = \frac{1}{2}(35)(30)\sin 50$ $= 402{\cdot}1 \ldots = 402$ [m^2] [nearest m^2] Area Field $2 = \frac{1}{3} \times$ Area Field 1 [*since both have common perp. height*] $= \frac{402{\cdot}1\ldots}{3} = 134$ [m^2] [nearest m^2] **OR** Total area $= \frac{1}{2}(35)(40)\sin 50 = 536{\cdot}2 \ldots$ So, area Field $2 = 536{\cdot}2 \ldots - 402{\cdot}1 \ldots$ $= 134$ [m^2] [nearest m^2]	**Scale 10D (0,3,5,8,10)** *Low Partial Credit* • Work of merit, for example, some correct substitution into formula for area of a triangle *Mid Partial Credit* • Finds the area of Field 1 or total area • Error(s) in finding the area of Field 1, but correctly finds the area of Field 2 from this *High Partial Credit* • Finds the area of Field 1, **and** either establishes the ratio relationship in area between both fields **or** forms the expression for the total area
(b)	$\lvert CB\rvert^2 = 35^2 + 30^2 - 2(35)(30)\cos 50$ $= 775{\cdot}14 \ldots$ $\lvert CB\rvert = \sqrt{775{\cdot}14 \ldots} = 27.8 \ldots = 28$ [m] [$\in \mathbb{N}$] Perimeter $= 35 + 30 + 28 = 93$ [m] [$\in \mathbb{N}$]	**Scale 10C (0,3,7,10)** *Low Partial Credit* • Some correct substitution into Cosine Rule *High Partial Credit* • Fully substituted Cosine Rule *Full Credit –1* • Finds $\lvert CB\rvert$ but fails to find the perimeter

Q9	Model Solution – 50 Marks	Marking Notes
(c) (i)	$\lvert P_1O\rvert = \sqrt{10^2 + 10^2} = 14{\cdot}142 \ldots$ km $\approx 14\,142$ m $T = \frac{14\,142}{343} = 41{\cdot}2 \ldots$ secs $= 41$ [secs] [$\in \mathbb{N}$] **OR** $\cos 45° = \frac{10}{\lvert P_1O\rvert}$ So $\lvert P_1O\rvert = \frac{10}{\cos 45°} = 14{\cdot}142 \ldots$ km, etc.	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, some correct substitution into a relevant formula, or a correct relevant conversion, or indicates a relevant measurement on the diagram, or 343×41 *High Partial Credit* • Finds $\lvert P_1O\rvert$ in metres • Error in finding $\lvert P_1O\rvert$, but continues to find T *Full Credit –1* • Finds T, but doesn't round appropriately or give conclusion • Calculator in incorrect mode, otherwise correct
(c) (ii)	$\lvert P_1P_2\rvert = 41 \times 255 = 10\,455$ m So $\tan\theta = \frac{455}{10\,000}$ $\theta = 2{\cdot}60 \ldots = 2{\cdot}6$[°] [1 D.P.]	**Scale 10C (0,3,7,10)** Note: accept use of 41 seconds or more accurate value from (c)(i) *Low Partial Credit* • Work of merit, for example, some correct substitution into a relevant formula *High Partial Credit* • Finds $\lvert P_1P_2\rvert$ • Error in finding $\lvert P_1P_2\rvert$, but finishes correctly
(d) (i)	For example: The time taken for the sound to go from P_3 to **O** [*LHS*] is the same as the time taken for the plane to go from P_3 to P_4 [*RHS*]	**Scale 5B (0,2,5)** *Partial Credit* • Work of merit, for example, implies LHS and / or RHS bears a relationship with time, or makes a relevant connection to distance or speed

Q9	Model Solution – 50 Marks	Marking Notes
(d) (ii)	$\frac{\sqrt{100+d^2}}{0{\cdot}343} = \frac{2d}{0{\cdot}255}$ $\frac{2\times 0{\cdot}343\times d}{0{\cdot}255} = \sqrt{100+d^2}$ $\frac{686}{255}d = \sqrt{100+d^2}$ $(7{\cdot}23\ldots)d^2 = 100 + d^2$ $(6{\cdot}23\ldots)d^2 = 100$ $d^2 = \frac{100}{6{\cdot}23\ldots} = 16{\cdot}03\ldots$ $d = \sqrt{16{\cdot}03\ldots} = 4{\cdot}00\ldots = 4$ [1 D.P.]	**Scale 10D (0,3,5,8,10)** **1.** Squares both sides and expands **2.** Writes $a \times d^2 = b$, for $a, b \in \mathbb{R}$ **3.** Find d *Low Partial Credit* • Work of merit, for example, some correct rearrangement, or indicates squaring *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$z = \frac{240-225}{12} = \frac{15}{12} = 1{\cdot}25$ $P(x > 240) = P(z > 1{\cdot}25)$ $= 1 - P(z < 1{\cdot}25)$ $= 1 - 0{\cdot}8944$ $= 0{\cdot}1056$ Answer: $10{\cdot}56\%$	**Scale 10D (0,3,5,8,10)** **1.** Find $\boldsymbol{z}$-score **2.** Find $\mathbf{0{\cdot}8944}$ **3.** Find solution *Low Partial Credit* • Work of merit, for example, some correct substitution into relevant formula, relevant diagram drawn, indicates μ or σ *Mid Partial Credit* • Finds z-score $\left(\frac{240-225}{12}\right)$ *High Partial Credit* • Finds z-score **and** further work, for example, finds $0{\cdot}8944$, or indicates $1 - P(x < 240)$ or similar
(a) (ii)	Look up $P = 0{\cdot}8$: $z = 0{\cdot}84$ or $0{\cdot}85$ Time $= \frac{225-x}{12} = 0 \cdot 84$ So Time$= 225 - 0{\cdot}84(12) = 214{\cdot}92$ Or Time $= 225 - 0{\cdot}85(12) = 214{\cdot}8$ Accept time $= 214$ [secs] or 215 [secs]	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, some correct substitution into relevant formula, relevant diagram drawn, indicates μ or σ *High Partial Credit* • Finds z-score **and** further work, for example, some correct substitution into relevant formula, or relevant diagram drawn
(b)	$1 - 0{\cdot}05 = 0{\cdot}95$ $P = 0{\cdot}95 \times 0{\cdot}95 \times 0{\cdot}95 \times 0{\cdot}05$ $= 0{\cdot}04286 \ldots = 0{\cdot}0429$ [4 D.P.]	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, $1 - 0{\cdot}05$, or some correct substitution into a relevant formula, or lists *NNNY* or similar *High Partial Credit* • Fully substituted correct formula • One probability missing or extra, and evaluates • Swaps $0{\cdot}05$ and $0{\cdot}95$, otherwise correct

Q10	Model Solution – 50 Marks	Marking Notes
(c)	$P(\text{at most } 2) = P(0 \text{ or } 1 \text{ or } 2)$ $= P(0) + P(1) + P(2)$ $= 0{\cdot}9^{20} + \binom{20}{1} 0{\cdot}1^1 0{\cdot}9^{19} + \binom{20}{2} 0{\cdot}1^2 0{\cdot}9^{18}$ $= 0{\cdot}12157 \ldots + 0{\cdot}27017 \ldots + 0{\cdot}28517 \ldots$ $= 0{\cdot}67692 \ldots = 0{\cdot}6769$ [4 D.P.]	**Scale 10D (0,3,5,8,10)** *Low Partial Credit* • 1st line of solution • Finds $0{\cdot}9$ *Mid Partial Credit* • Fully substituted formulae for **two** of $P(0)$, $P(1)$, and $P(2)$ *High Partial Credit* • Fully substituted formulae for $P(0)$, $P(1)$, and $P(2)$
(d)	50 possible pairs of numbers add to 101: $1 + 100, \;\; 2 + 99, \ldots, \;\; 50 + 51$ $300C2 = 44\,850$ pairs in total. So $P = \frac{50}{44\,850} = \frac{1}{897}$ **OR** 100 different $1st$ numbers could be picked; for each, only one $2nd$ number will give 101: $P = \frac{100}{300} \times \frac{1}{299} = \frac{1}{897}$	**Scale 5C (0,2,3,5)** *Low Partial Credit* • Work of merit, for example, indicates at least two possible pairs adding to 101, or $300C2$ *High Partial Credit* • Finds 50 **and** puts over a relevant number • Finds 50 **and** C_2^{300} • $\frac{100}{300} \times \frac{1}{299}$

Q10	Model Solution – 50 Marks	Marking Notes
(e)	**Windy**: $\frac{5265}{6000} = 0{\cdot}8775$ So $z = 1{\cdot}16$ or $1{\cdot}17$ So Time $= 254 + 1{\cdot}16(38) = 298{\cdot}08$ Or Time $= 254 + 1{\cdot}17(38) = 298{\cdot}46$ **Sunny**: $z = \frac{298{\cdot}08 - 247}{29} = 1{\cdot}76137\ldots$ Or $z = \frac{298{\cdot}46 - 247}{29} = 1{\cdot}77448\ldots$ Taking $z = 1{\cdot}76$, there are $0{\cdot}9608$ of runners are faster than Sorcha, so position $= 0{\cdot}9608 \times 2000 = 1921{\cdot}6$, i.e. 1921th or 1922th Taking $z = 1{\cdot}77$, position $= 0{\cdot}9616 \times 2000 = 1923{\cdot}2$, i.e. 1923th or 1924th Taking $z = 1{\cdot}78$, position $= 0{\cdot}9625 \times 2000 = 1925$th	**Scale 15E (0, 3, 6, 9, 12, 15)** **1.** Finds $\frac{5265}{6000}$ **2.** Finds the z-score for the Windy marathon **3.** Finds the time for the marathons **4.** Finds the z-score for the Sunny marathon **5.** Finds the finishing position *Low Partial Credit* • Work of merit, for example, draws a relevant diagram, indicates μ or σ *Low Mid Partial Credit* • 2 steps correct *High Mid Partial Credit* • 3 steps correct *High Partial Credit* • 4 steps correct