Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2022

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Leaving Certificate 2022

Mathematics

Higher Level

Paper 1

Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	
10-mark scale			0, 3, 7, 10	0, 3, 5, 8, 10
15-mark scale				0, 4, 8, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- response of no substantial merit (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. A penalty for an omitted unit is applied only once in each section (a), (b), (c) etc. There is no penalty for omitted units if the question specifies the unit to be used in the answer.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

In general, an answer without sufficient supporting work is awarded the lowest non-zero level of credit (typically *Partial Credit* or *Low Partial Credit*, as appropriate).

Summary of mark allocations and scales to be applied

Section A (150) **Answer any five questions**		**Section B (150)** **Answer any three questions**	
Question 1 (30)	**Question 4 (30)**	**Question 7 (50)**	**Question 9 (50)**
(a) 10C	(a) 10C	(a), (b) 10D	(a) 5B
(b) 5B	(b)(i) 10C	(c) 5C	(b) 10C
(c) 15D	(b)(ii) 10D	(d) 15D	(c) 5A
		(e) 10C	(d) 10C
Question 2 (30)	**Question 5 (30)**	(f) 5C	(e) 5B
(a) 5C	(a) 5B	(g) 5B	(f)(i) 10C
(b)(i) 10D	(b)(i) 15D		(f)(ii) 5C
(b)(ii) 15D	(b)(ii) 10D	**Question 8 (50)**	
		(a), (b) 15D	**Question 10 (50)**
Question 3 (30)	**Question 6 (30)**	(c) 5C	(a) 5B
(a)(i) 5C	(a) 15D	(d) 10C	(b) 10D
(a)(ii) 10D	(b) 10D	(e) 10D	(c) 10C
(a)(iii) 5C	(c) 5C	(f) 10D	(d) 10C
(b) 10D			(e)(i) 10C
			(e)(ii) 5C

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
	Tick	Work of relevance	The work presented in the body of the script merits full credit
	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
	P		The work presented in the body of the script merits *Partial Credit*
	L		The work presented in the body of the script merits *Low Partial Credit*
	M		The work presented in the body of the script merits *Mid Partial Credit*
	H		The work presented in the body of the script merits *High Partial Credit*
	F star		The work presented in the body of the script merits *Full Credit – 1*
	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
	Oversimplify	The candidate has oversimplified the work	
	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and  and  appear in the body of the work, then  should be placed in the right margin.

In the case of a **D scale** with the same annotations,  should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Where "work of merit" is referred to in the Marking Notes, example(s) are given to demonstrate the standard of work that is to be considered work of merit in that particular question.

Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. It is specifically noted where this does not hold. Note also that these steps may not need to be presented in the order specified in the Marking Notes.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$b^2 - 4ac = 0$ $m^2 - 4(3)(3) = 0$ $m^2 = 36$ $m = \pm 6$ **OR** $(3x-3)(x-1) = 0$ so $m = 6$ Or $(3x+3)(x+1) = 0$ so $m = -6$ **OR** Differentiates: $6x - m = 0$ So $x = \frac{m}{6}$ $3\left(\frac{m^2}{36}\right) - \frac{m^2}{6} + 3 = 0$ $m^2 = 36$ $\boldsymbol{m = \pm 6}$ **OR** $x^2 - \frac{m}{3}x + 1 = 0$ Equal roots, α and α: $2\alpha = \frac{m}{3}$ so $\alpha = \frac{m}{6}$ $\alpha^2 = 1$ so $\frac{m^2}{36} = 1$ So $m = \pm 6$	**Scale 10C (0, 3, 7, 10)** **3 steps:** **1.** $b^2 - 4ac = 0$ **2.** Substitutes in for a, b, c **3.** Solves for m *Low Partial Credit* • Work of merit, for example, identifies a or b or c • Some correct differentiation • Mentions perfect square • Divides the given equation by 3 *High Partial Credit* • Two steps correct • One correct value of m found • Finds $x = \frac{m}{6}$ and substitutes into function • Finds correct factors or roots

Q1	Model Solution – 30 Marks	Marking Notes
(b)	$(2x+3)^2 \geq 0$ for all $x \in \mathbb{R}$ So $(2x+3)^2 + 7 \geq 7 > 0$ **OR** $(2x+3)^2 = -7$ $2x+3 = \pm\sqrt{-7}$, which is not real **OR** $4x^2 + 12x + 9 + 7 = 0$ $4x^2 + 12x + 16 = 0$ $b^2 - 4ac = 12^2 - 4(4)(16) < 0$, so no real roots **OR** The graph of $y = (2x+3)^2 + 7$ is U-shaped and the minimum value of y is 7, therefore, no real solutions.	**Scale 5B (0, 2, 5)** *Partial Credit* • Work of merit, for example, mentions that a squared number is non-negative, or $b^2 - 4ac < 0$ • Correct transposition of 7 • Some correct work in the expansion of $(2x+3)^2$

Q1	Model Solution – 30 Marks	Marking Notes
(c)	**(i)** $3(-1)^2 + 2(-1) + 5$ $= 3 - 2 + 5 = 6 \neq 0$ **(ii)** $(x+1)(ax+b) + c$ $= ax^2 + (a+b)x + b + c = 3x^2 + 2x + 5$ So $a = 3$, $3 + b = 2$, so $b = -1$ $-1 + c = 5$, so $c = 6$. **OR** $3x - 1$ $x+1 \quad \sqrt{3x^2 + 2x + 5}$ $\underline{3x^2 + 3x}$ $-x + 5$ $\underline{-x - 1}$ 6 Remainder $= 6$ **OR** $3x$ | -1 x | $3x^2$ | $-x$ 1 | $3x$ | -1 So remainder $= 5 - (-1) = 6$	**Scale 15D (0, 4, 8, 12, 15)** Note: (i) may be done by long division, so "substantial work of merit" for (ii) may appear in (i). In (ii) accept remainder = candidate's $f(-1)$ in (i) *Low Partial Credit* • Work of merit, for example, in (i): substitutes in $x = -1$, or mentions $x + 1$; Substitutes into quadratic formula in (ii): some expansion of given product, or some correct division, or sets up long division correctly *Mid Partial Credit* • (i) correct and work of merit in (ii) • Substantial work of merit in (ii), for example, finds $-x + 5$ in long division method, or finds two of a, b, and c correctly *High Partial Credit* • (i) correct and substantial work of merit in (ii) • (ii) correct *Full Credit -1* • (i) correct and correct long division in (ii), but answer (remainder) not identified

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$\frac{2x^3}{3}+\frac{5x^2}{2}+6x+c$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Some correct integration *High Partial Credit* • 3 of the 4 terms correct (including $+c$ as a term) *Full Credit -1* • $+c$ term missing
(b) (i)	$\int_0^2(ax^2+bx+c)\,dx=538$ $\frac{ax^3}{3}+\frac{bx^2}{2}+cx\Big\vert_{x=0}^{x=2}=538$ $\frac{a(2^3)}{3}+\frac{b(2^2)}{2}+c(2)=538$ $4a+3b+3c=807$	**Scale 10D (0, 3, 5, 8, 10)** Note: integration is required in order to be awarded any credit **4 steps:** **1.** $\int(ax^2+bx+c)dx$ **2.** Integrates all 3 terms **3.** Subs in limits (accept without 0 subbed in) and sets expression equal to 538 **4.** Simplifies to given expression *Low Partial Credit* • Work of merit, for example, integration indicated *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q2	Model Solution – 30 Marks	Marking Notes
(b) (ii)	**(2) − (1):** $24a + 6b = 72$ …**(4)** **(3) − (2)**: $48a + 6b = -216$ …**(5)** **(5) − (4)**: $24a = -288$ So $a = -12$ **(4)**: $24(-12) + 6b = 72$ $6b = 360$ so $b = 60$ **(1):** $4(-12) + 3(60) + 3c = 807$ $3c = 675$ so $c = 225$ **OR** Eq1: $$a = \frac{807 - 3b - 3c}{4}$$ Eq2: $$7(807 - 3b - 3c) + 9b + 3c = 879$$ $$c = \frac{795 - 2b}{3}$$ Eq3: $$19\big(807 - 3b - (795 - 2b)\big) + 15b + 795 - 2b = 663$$ $$-6b = -360$$ $$b = 60$$ Back substitution: $$c = \frac{795 - 2(60)}{3}$$ $$c = 225$$ $$a = \frac{807 - 3(60) - 3(225)}{4}$$ $$a = -12$$	**Scale 15D (0, 4, 8, 12, 15)** *Method 1:* **3 steps:** **1.** Two equations in the same 2 variables **2.** One equation in 1 variable **3.** Finds 3 variables *Method 2:* **1.** Writes one variable in terms of the other two **2.** Writes a second variable in terms of **ONE** of the other variables **3.** Finds 3 variables *Low Partial Credit* • Work of merit, for example, $4a = 807 - 3b - 3c$ *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

Q3	Model Solution – 30 Marks	Marking Notes
(a) (i)	$z - iz = 6 + 2i - i(6 + 2i)$ $= 6 + 2i - 6i - 2i^2$ $= 8 - 4i$ **OR** $z(1 - i) = 8 - 4i$ $z = \frac{8-4i}{1-i}$ $z = \frac{(8-4i)(1+i)}{(1-i)(1+i)}$ $z = \frac{12+4i}{2} = 6 + 2i$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, some correct substitution, or $z(1 - i)$ *High Partial Credit* • Mishandles $-2i^2$, otherwise correct • Multiplies numerator and denominator by conjugate
(a) (ii)	$\|z\|^2 = 6^2 + 2^2 = 40$ $\|iz\|^2 = 2^2 + 6^2 = 40$ $\|z - iz\|^2 = 8^2 + 4^2 = 80$ $40 + 40 = 80$ **OR** $\|x + iy\|^2 + \|-y + xi\|^2 = \|(x + y) + (y - x)i\|^2$ $x^2 + y^2 + y^2 + x^2$ $= x^2 + 2xy + y^2 + x^2 - 2xy + y^2$ $2x^2 + 2y^2 = 2x^2 + 2y^2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* • Work of merit, for example, correct formula with some substitution *Mid Partial Credit* • Two correct values found, from $\|z\|$, $\|iz\|$ and $\|z - iz\|$ *High Partial Credit* • Two correct values found, from $\|z\|^2$, $\|iz\|^2$, and $\|z - iz\|^2$ *Full Credit –1* • Finds $\|z\|^2$, $\|iz\|^2$, and $\|z - iz\|^2$, but no conclusion of equality
(a) (iii)	Radius $= \sqrt{80} \div 2 = \sqrt{20}$ Area $= \pi\left(\sqrt{20}\right)^2 = 20\pi$ square units **OR** Centre $= \frac{6+2i-2+6i}{2} = 2 + 4i$ Radius $= \sqrt{(6-2)^2 + (2-4)^2} = \sqrt{20}$ Area $= \pi\left(\sqrt{20}\right)^2 = 20\pi$ square units (Accept without units)	**Scale 5C (0, 2, 3, 5)** Allow solution treating problem as being in the real 2D co-ordinate plane rather than the complex plane *Low Partial Credit* • Some work of merit, for example, some substitution into a relevant formula *High Partial Credit* • Finds radius • Some work of merit in finding radius, and finds area based on incorrect radius

Q3	Model Solution – 30 Marks	Marking Notes
(b)	$\tan A = \frac{1}{\sqrt{3}}$, so $A = 30°$, so $\theta = 330°$ $r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$ $\left(\sqrt{3} - i\right)^9 = 2^9(\cos 9(330) + i \sin 9(330))$ $= 512(\cos 2970 + i \sin 2970)$ $= 0 + 512i$ $a = 0,\ b = 512$	**Scale 10D (0, 3, 5, 8, 10)** Note: polar form must be used to achieve any credit Note: Accept $0 + 512i$ for *Full Credit* Note that argument may also be given as $\theta = -30°$, etc., or $\theta = \frac{11\pi}{6}$, etc. **4 steps:** **1.** Finds r **2.** Finds θ **3.** Subs into de Moivre's Theorem **4.** Evaluates *Low Partial Credit* • Work of merit, for example, plots $\sqrt{3} - i$, or some correct substitution into de Moivre's Theorem *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • a and b not explicitly stated and solution given as $512i$

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$u_3 = \sqrt{\frac{u_2}{u_1}} = \sqrt{\frac{64}{2}} = \sqrt{32} = (2^5)^{\frac{1}{2}} = 2^{\frac{5}{2}}$	**Scale 10C (0, 3, 7, 10)** **3 steps:** **1.** Substitutes u_1 and u_2 into u_3 **2.** Writes 64 or 32 as a power of 2 **3.** Finishes (deals with square root) *Low Partial Credit* • Work of merit, for example, some correct substitution into u_3 *High Partial Credit* • 2 steps correct
(b) (i)	$[5e^k - 13 = 13 - 5e^{-k}\,]$ $5y - 13 = 13 - \frac{5}{y}$ $5y^2 - 13y = 13y - 5$ $5y^2 - 26y + 5 = 0$ **OR** $T_2 - T_1 = T_3 - T_2$ $T_1 + T_3 = 2T_2$ $5(e^k)^2 - 26(e^k) + 5 = 0$ $5e^{2k} - 26e^k + 5 = 0$ $5e^k + 5e^{-k} = 26$ $T_1 + T_3 = 2T_2$ **OR** $a = \frac{5}{y}$ $\frac{5}{y} + d = 13$ $d = 13 - \frac{5}{y}$ $13 + \left(13 - \frac{5}{y}\right) = 5y$ $26y - 5 = 5y^2$ $5y^2 - 26y + 5 = 0$	**Scale 10C (0, 3, 7, 10)** Each method shown has 3 steps. *Method 1:* **1.** Equates common differences **2.** Replaces e^k with y **and** e^{-k} with $\frac{1}{y}$ or y^{-1} **3.** Writes in required form *Method 2:* **1.** Shows $T_1 + T_3 = 2T_2$ for any arithmetic sequence **2.** Replaces y with e^k and simplifies **3.** Divides by e^k to show $T_1 + T_3 = 2T_2$ *Method 3:* **1.** Finds the common difference in terms of y **2.** Finds equation in y **3.** Writes in required form *Low Partial Credit* • Work of merit, for example, finds one common difference, or replaces e^k with y **or** y with e^k, or states $T_3 - T_2 = T_2 - T_1$ *High Partial Credit* • 2 steps correct

Q4	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$(y-5)(5y-1)=0$ $y=5$ or $\frac{1}{5}$ $e^k=5$ or $e^k=\frac{1}{5}$ $k=\ln 5$ or $k=\ln\frac{1}{5}=\ln(5^{-1})=-\ln 5$	**Scale 10D (0, 3, 5, 8, 10)** **3 steps:** **1.** Fully substituted quadratic formula **OR** factors found **2.** Solves for y **3.** Solves for k, in correct form *Low Partial Credit* • Work of merit, for example, effort at factorisation, or identifies a, b, or c *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct *Full Credit –1* • 2 values of e^k correctly found, only 1 value of k correctly found

Q5	Model Solution – 30 Marks	Marking Notes
(a)	$g'(x) = 2x - (-1x^{-2})$ $= 2x + \frac{1}{x^2}$ **OR** $g'(x) = 2x - \frac{x(0)-1(1)}{x^2} = 2x + \frac{1}{x^2}$	**Scale 5B (0, 2, 5)** Note: No credit if differentiation is not used. Accept the answer in an unsimplified form, for example, $2x - (-1x^{-2})$ *Partial Credit* • Some correct differentiation
(b) (i)	$2x^2 - 23x + 63$ $x+1 \quad \sqrt{2x^3 - 21x^2 + 40x + 63}$ $\underline{2x^3 + 2x^2}$ $-23x^2 + 40x + 63$ $\underline{-23x^2 - 23x}$ $63x + 63$ $\underline{63x + 63}$ 0 $(x+1)(2x^2 - 23x + 63) = 0$ $(x+1)(2x-9)(x-7) = 0$ $x = -1$, 4·5, or 7 **OR** $(x+1)(2x^2 - 23x + 63) = 0$ $(x+1)(2x-9)(x-7) = 0$ $x = -1$, 4·5, or 7	**Scale 15D (0, 4, 8, 12, 15)** Accept correct answers, with 4·5 and 7 verified, for *Full Credit*. **4 steps:** **1.** Finds 1 term in $2x^2 - 23x + 63$ **2.** Finds remaining 2 terms **3.** Factorises $2x^2 - 23x + 63$ **OR** fully substituted formula **4.** Finds 3 solutions *Low Partial Credit* • Work of merit, for example, some correct division, or sets up long division correctly, or sets $x + 1 = 0$ • Correct answers with no work *Mid Partial Credit* • 2 steps correct • Two roots correct with sufficient work (note: supporting work is not needed for $x = -1$) *High Partial Credit* • 3 steps correct

	$2x^2$	$-23x$	63
x	$2x^3$	$-23x^2$	$63x$
1	$2x^2$	$-23x$	63

Q5	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$f'(x) = 6x^2 - 42x + 40 < 0$ Roots: $\frac{42 \pm \sqrt{42^2 - 4(6)(40)}}{2(6)} = \frac{42 \pm \sqrt{804}}{12}$ $= 1{\cdot}137 \ldots$ or $= 5{\cdot}862 \ldots$ So: $1{\cdot}14 < x < 5{\cdot}86$ [2 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** Note: in general, solution must be based on differentiation in order for any credit to be awarded Accept $1{\cdot}14 \leq x \leq 5{\cdot}86$ **4 steps:** **1.** Finds $f'(x)$ **2.** Fully substituted formula **3.** Finds roots **4.** Finds correct range of values *Low Partial Credit* • Some correct differentiation *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2(x+h)^2+4(x+h)-2x^2-4x}{h}$ $=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+4x+4h-2x^2-4x}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2+4h}{h}$ $=\lim_{h\to 0}(4x+2h+4)=4x+4$	**Scale 15D (0, 4, 8, 12, 15)** Note: no credit unless differentiation from first principles is being used. **4 steps:** **1.** Finds $f(x+h)$ **2.** Finds $f(x+h)-f(x)$ **3.** Finds $\frac{f(x+h)-f(x)}{h}$ **4.** Finds $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ *Low Partial Credit:* • Work of merit, for example, some substitution into $f(x+h)$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct • Left hand side missing, otherwise correct

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$A = l \times w = 4x \times x = 4x^2$ When $A = 225,\ 4x^2 = 225,$ so $x^2 = \frac{225}{4}$, i.e. $x = \frac{15}{2}$ $\frac{dA}{dx} = 8x = 8\left(\frac{15}{2}\right) = 60$ cm²/cm Accept without unit **OR** $l = 4x$ so $x = \frac{l}{4}$ $A = l \times \frac{l}{4} = \frac{1}{4}l^2$ $\frac{dl}{dx} = 4$ and $\frac{dA}{dl} = \frac{1}{2}l$ $\frac{dA}{dx} = \frac{dA}{dl} \times \frac{dl}{dx}$ $\frac{dA}{dx} = \frac{1}{2}l \times 4$ When A = 225, $\frac{1}{4}l^2 = 225$, so $l = 30$ $\frac{dA}{dx} = \frac{1}{2}(30)(4) = 60$ cm²/cm Accept without unit	**Scale 10D (0, 3, 5, 8, 10)** **Method 1:** **1.** Write A in terms of x **2.** Find $x = \frac{15}{2}$ **3.** Find $\frac{dA}{dx}$ **4.** Evaluate $\frac{dA}{dx}$ at $x = \frac{15}{2}$ *Low Partial Credit:* • Work of merit, for example, $l = 4x$; mentions $\frac{dA}{dx}$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct **Method 2:** *Low Partial Credit*: • States relevant derivative, for example, $\frac{dA}{dx}$ • $l = 4x$ *Mid Partial Credit*: • Any **two** of the following: ○ $\frac{dl}{dx} = 4$ ○ $\frac{dA}{dl} = \frac{1}{2}l$ ○ $\frac{dA}{dx} = \frac{dA}{dl} \times \frac{dl}{dx}$ ○ $l = 30$ *High Partial Credit:* • $\frac{dA}{dx} = \frac{dA}{dl} \times \frac{dl}{dx}$, **and** any **two** others from the MPC list above

Q6	Model Solution – 30 Marks	Marking Notes
(c)	*Quadratic graph through* $(0,-3)$*,* $(1,0)$*,* $(2,1)$*,* $(3,0)$*, and* $(4,-3)$*.*	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, one correct point with integer co-ordinates plotted, or correct shape *High Partial Credit:* • Three correct points with integer co-ordinates plotted *Full Credit –1:* • Point at $x = 4$ plotted incorrectly, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(a) **(b)**	**(a)** $h(4) = 2(4^3) - 28 \cdot 5(4^2) + 105(4) + 70$ $= 162$ BPM **(b)** $h'(x) = 3(2x^2) - 2(28 \cdot 5x) + 105$ $= 6x^2 - 57x + 105$	**Scale 10D (0, 3, 5, 8, 10)** Accept unsimplified version of $h'(x)$ *Low Partial Credit* • Work of merit in either part, for example, in (a), some correct substitution; in (b): some correct differentiation *Mid Partial Credit* • (a) or (b) correct • Work of merit in both (a) and (b) *High Partial Credit* • One part correct and work of merit in the other *Full Credit –1* • All correct, except that unit is missing or incorrect in (a)
(c)	$h'(2) = 6(2^2) - 57(2) + 105 = 15$ Explanation: It is the rate at which Hannah's heart rate is increasing after / at 2 minutes.	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, some correct substitution into $h'(x)$, or explanation shows some understanding of derivative as rate of change *High Partial Credit* • One part correct ($h'(2)$ or explanation)

Q7	Model Solution – 50 Marks	Marking Notes
(d)	Least value of $h(x) = h(0) = 70$ [from graph] $h'(x) = 6x^2 - 57x + 105 = 0$ at local max $2x^2 - 19x + 35 = 0$ $(2x - 5)(x - 7) = 0$ $x = 2{\cdot}5$ or 7 Max $= h(2{\cdot}5)$ [from graph] $= 2(2 \cdot 5^3) - 28 \cdot 5(2 \cdot 5^2) + 105(2 \cdot 5) + 70$ $= 185{\cdot}625$	**Scale 15D (0, 4, 8, 12, 15)** **4 steps:** **1.** Finds least value of $h(x)$ **2.** Sets $h'(x) = 0$ **3.** Finds x-value for max **4.** Finds greatest value of $h(x)$ *Low Partial Credit* • Work of merit, for example, indicates least value at $h(0)$ • Any correct differentiation • Brings down $h'(x)$ from (b) *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct
(e)	$h'(x) = 6x^2 - 57x + 105$ Decreasing most quickly at $h''(x) = 0$ So $12x - 57 = 0$ So $x = 4{\cdot}75$ minutes $= 4$ mins 45 secs **OR** Decreasing most quickly at midpoint of local max/min, that is, $x = \frac{2{\cdot}5+7}{2} = 4{\cdot}75$ minutes $= 4$ mins 45 secs	**Scale 10C (0, 3, 7, 10)** **3 steps** (note that step 1 must be done or implied, in order for steps 2 or 3 to be considered correct): **1.** Indicates $h''(x) = 0$ **OR** midpoint of local max and min values **2.** Finds value of x in decimal / fraction form **3.** Finds x in required from *Low Partial Credit* • Work of merit, for example, marks relevant point on graph, indicates $h''(x)$ • Brings down $h'(x)$ from (b) • First differences found **and** some indication of the greatest first difference *High Partial Credit* • 2 steps correct, including Step 1

Q7	Model Solution – 50 Marks	Marking Notes
(f)	$b'(x) = h'(x)$ $k'(x) = 0{\cdot}9\ h'(x)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit in 1 part, for example, $b(x) = h(x) + 15$, or $k(x) = 0{\cdot}9\ h(x)$ *High Partial Credit* • 1 part correct
(g)	$m(x) = 2(0 \cdot 8x)^3 - 28{\cdot}5(0 \cdot 8x)^2 + 105(0 \cdot 8x) + 70$ $= 1{\cdot}024x^3 - 18{\cdot}24x^2 + 84x + 70$	**Scale 5B (0, 2, 5)** *Partial Credit* • Work of merit, for example, expands $(0 \cdot 8x)^2$ or $(0 \cdot 8x)^3$, or some substitution of $0 \cdot 8x$ into $h(x)$

Q8	Model Solution – 50 Marks	Marking Notes
(a), (b)	**(a)** $12 \quad 42 \quad 102 \quad 132 \quad 102 \quad 42 \quad 12 \quad 42 \quad 102$ **(b)** *Appropriate graph through the 9 points*	**Scale 15D (0, 4, 8, 12, 15)** 17 items are required: 8 table entries and 9 plots (which need to be joined appropriately for *Full Credit*) *Low Partial Credit* • Any 1 item correct *Mid Partial Credit* • Any 8 items correct *High Partial Credit* • Any 12 items correct *Full Credit –1* • All items correct but points not joined or joined inappropriately • All items but 1 correct, and points appropriately joined
(c)	Period $= 6$ Range $= [12, 132]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, mentions 12, 132, or 120; or period or range clearly marked on graph *High Partial Credit* • Period or range correct *Full Credit –1* • Period and range correct, but swapped • Period $= 6$ and Range $= 120$ • Range $= [12, 132]$ and Period given as 0 – 6 or 1 – 7, etc.

Q8	Model Solution – 50 Marks	Marking Notes
(d)	$50 \div 6 = 8$ periods, 2 minutes 1 period: 4 mins above 42 m 8 periods: $8 \times 4 = 32$ mins above 42 m 50 mins: 32 + 2 = 34 mins above 42 m	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, $50 \div 6$, or $6 + 6 + \cdots$, or indicates 4 • The line $h(t) = 42$ shown on the graph *High Partial Credit* • Finds 32 mins • Adds 2 to a relevant number of mins
(e)	$60\cos\left(\frac{\pi}{3}t\right) = 72 - 110 = -38$ $\cos\left(\frac{\pi}{3}t\right) = -\frac{38}{60}$ Ref angle $= \cos^{-1}\left(\frac{38}{60}\right) = 0{\cdot}8849 \ldots$ radians 1st time: Quadrant 2 2nd time: Quadrant 3 $\frac{\pi}{3}t_2 = \pi + 0{\cdot}8849 \ldots$ radians $t = \frac{3(\pi + 0{\cdot}8849\ldots)}{\pi} = 3{\cdot}845 \ldots = 3{\cdot}85$ mins [2 DP] **OR** $\cos\left(\frac{\pi}{3}t\right) = -\frac{38}{60}$ $\frac{\pi}{3}t_1 = \cos^{-1}\left(-\frac{38}{60}\right)$ $t_1 = 2.15494 \ldots$ $t = 6 - 2.15494$ $t = 3.85$ mins	**Scale 10D (0, 3, 5, 8, 10)** **1.** Isolates $cos\left(\frac{\pi}{3}t\right)$ **2.** Finds reference angle or $\frac{\pi}{3}t_1$ **3.** Finds $\frac{\pi}{3}t_2$ or t_1 **4.** Finds t *Low Partial Credit* • Work of merit, for example, some correct work towards isolating $\cos\left(\frac{\pi}{3}t\right)$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Calculator in degree mode, otherwise correct

Q8	Model Solution – 50 Marks	Marking Notes
(f)	$\frac{1}{8}\int_0^8 \left(72 - 60\cos\left(\frac{\pi}{3}t\right)\right) dt$ $= \frac{1}{8}\left(72t - \frac{3\times 60\sin\left(\frac{\pi}{3}t\right)}{\pi}\right)_{t=0}^{t=8}$ $= \frac{1}{8}\left(72(8) - \frac{180\sin\frac{8\pi}{3}}{\pi} - \left(-\frac{180\sin 0}{\pi}\right)\right)$ $= 72 - 6{\cdot}20 \ldots = 65{\cdot}8$ m [1 DP]	**Scale 10D (0, 3, 5, 8, 10)** Note: integration is required in order to be awarded any credit **4 steps** (if $\frac{1}{8}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct): **1.** $\frac{1}{8}\int_0^8 h(t)\, dt$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit* • Work of merit, for example, integration indicated *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Calculator in degree mode, otherwise correct

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$15(0 \cdot 6^{2 \cdot 5}) = 4 \cdot 182 \ldots = 4 \cdot 18$ [mg] [2 DP]	**Scale 5B (0, 2, 5)** Accept correct answer without work *Low Partial Credit* • Work of merit, for example, correct substitution into given expression
(b)	$15(0 \cdot 6^t) = 1$ $0 \cdot 6^t = \frac{1}{15}$ $\ln 0 \cdot 6^t = t \ln 0 \cdot 6 = \ln \frac{1}{15}$ $t = \frac{\ln \frac{1}{15}}{\ln 0 \cdot 6} = 5 \cdot 30 \ldots = 5 \cdot 3$ [days] **OR** $t = \log_{0.6} \frac{1}{15} = 5.30 \ldots$ $= 5.3$ [days] [1 DP]	**Scale 10C (0, 3, 7, 10)** **1.** Isolates $0 \cdot 6^t$ **2.** Converts to log equation (not necessarily to base e) **3.** Solves for t *Low Partial Credit* • Work of merit, for example, $15(0 \cdot 6^t) = 1$ • 5.3 days by trial and improvement *High Partial Credit* • 2 steps correct
(c)	15: amount from injection just given $15(0 \cdot 6)$: amount from injection 1 day ago $15(0 \cdot 6^2)$: amount from injection 2 days ago $15(0 \cdot 6^3)$: amount from injection 3 days ago **OR** There is 15 mg from the injection just given, and the amount from each previous injection has reduced by 40% each day	**Scale 5A (0, 5)**
(d)	$15 + 15(0 \cdot 6) + \cdots + 15(0 \cdot 6^9)$ $S_n = \frac{a(1-r^n)}{1-r} = \frac{15(1-0 \cdot 6^{10})}{1-0 \cdot 6}$ $= 37 \cdot 273 \ldots = 37 \cdot 27$ [mg] [2 DP]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, indicates sum of two or more relevant terms, identifies a or r, or S_n formula with some substitution *High Partial Credit* • S_n formula fully substituted

Q9	Model Solution – 50 Marks	Marking Notes
(e)	$S_\infty = \frac{a}{1-r} = \frac{15}{1-0{\cdot}6} = 37{\cdot}5$ [mg]	**Scale 5B (0, 2, 5)** *Partial Credit* • S_∞ formula with some substitution • Identifies a or r
(f) (i)	Amount immediately after n th injection: $d + d(0 \cdot 85) + \cdots + d(0 \cdot 85^{n-1})$ $= \frac{a(1-r^n)}{1-r} = \frac{d(1-0{\cdot}85^n)}{1-0{\cdot}85}$ $= \frac{20d(1-0{\cdot}85^n)}{3}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, identifies a or r, or one term in series (other than d) *High Partial Credit* • Fully correct substitution into formula • One error in substitution and finishes correctly
(f) (ii)	$\frac{20d(1-0{\cdot}85^7)}{3} = 50$ $d = \frac{50\times3}{20(1-0{\cdot}85^7)} = 11{\cdot}03 \ldots = 11$ [mg] $[\in \mathbb{N}]$	**Scale 5C (0, 2, 3, 5)** **2 steps:** **1.** Sets up equation in d **2.** Finds value of d *Low Partial Credit* • Work of merit, for example, some correct substitution into formula for amount of drug, or identifies a or r *High Partial Credit* • 1 step correct

Q10	Model Solution – 50 Marks	Marking Notes
(a)	$P(3) = 0{\cdot}82 - 0{\cdot}12\ln 4$ $= 0{\cdot}653 \ldots = 0{\cdot}65$ [2 DP]	**Scale 5B (0, 2, 5)** Accept correct answer without work Accept $65{\cdot}36\%$ or 65% *Partial Credit* • Work of merit, for example, indicates $P(3)$
(b)	$0{\cdot}82 - 0{\cdot}12\ln(t+1) = 0{\cdot}55$ $0{\cdot}12\ln(t+1) = 0{\cdot}27$ $\ln(t+1) = \frac{0{\cdot}27}{0{\cdot}12} = 2{\cdot}25$ $t+1 = e^{2{\cdot}25}$ $t = e^{2{\cdot}25} - 1 = 8{\cdot}487 \ldots = 8{\cdot}49$ [hours] [2 DP]	**Scale 10D (0, 3, 5, 8, 10)** Note: if logs are mishandled, award at most *Mid Partial Credit* **1.** Sets up equation **2.** Isolates $ln(t+1)$ **3.** Finds $t+1$ **4.** Finishes *Low Partial Credit* • Work of merit, for example, sets up equation correctly, or incorrect equation set up but some subsequent correct work • Correct answer using trial and improvement *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct
(c)	**(i)** $P'(t) = \frac{-0{\cdot}12}{t+1}$ $P'(1) = \frac{-0{\cdot}12}{1+1} = -0{\cdot}06$ **(ii)** The proportion decreases [as t goes up]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Some correct differentiation in (i) • (ii) correct *High Partial Credit* • (i) correct • (ii) correct and some correct differentiation in (i)

Q10	Model Solution – 50 Marks	Marking Notes
(d)	$[P''(t) = 0$ at all points of inflection$]$ $P''(t) = -0{\cdot}12(-(t+1)^{-2})$ $= \frac{0{\cdot}12}{(t+1)^2} = 0 \Rightarrow 0 \cdot 12 = 0$, contradiction. [So, no points of inflection] **OR** $P''(t) = \frac{0.12}{(t+1)^2} \neq 0$ [as $0{\cdot}12 \neq 0$] [So, no points of inflection]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Indicates second derivative *High Partial Credit* • Finds $P''(t)$ • Error in finding $P''(t)$, but shows that their $P''(t)$ can't be 0 in the given domain
(e) (i)	$\log_{10} A = \log_{10} B(t+1)^c$ $\log_{10} A = \log_{10} B + \log_{10}(t+1)^c$ $c \log_{10}(t+1) = \log_{10} A - \log_{10} B$ $c = \frac{\log_{10} A - \log_{10} B}{\log_{10}(t+1)}$ **OR** $(t+1)^c = \frac{A}{B}$ $\log_{t+1} \frac{A}{B} = c$ $c = \log_{t+1} A - \log_{t+1} B$ $c = \frac{\log_{10} A}{\log_{10}(t+1)} - \frac{\log_{10} B}{\log_{10}(t+1)}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, correct relevant equation involving logs • $(t+1)^c = \frac{A}{B}$ *High Partial Credit* • Correct equation involving $\log_{10} A$, $\log_{10} B$, and $\log_{10}(t+1)$ • Correct equation involving $\log_{t+1} A$ and $\log_{t+1} B$
(e) (ii)	$c = \frac{\log_{10} 80 - \log_{10} 47}{\log_{10}(24+1)} = 0{\cdot}1652 \ldots = 0{\cdot}165$ [3 DP] **OR** $80 = 47(24+1)^c$ $(24+1)^c = \frac{80}{47}$ $\ln 25^c = \ln \frac{80}{47}$ $c = \frac{\ln \frac{80}{47}}{\ln 25} = 0{\cdot}1652 \ldots = 0{\cdot}165$ [3 DP] **OR** $80 = 47(24+1)^c$ $(24+1)^c = \frac{80}{47}$ $c = \log_{25} \frac{80}{47} = 0{\cdot}1652 \ldots = 0{\cdot}165$ [3 DP]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, correctly substitutes in some value from (ii) to expression/equation in (i) [the one given or the one the candidate produces] • A and B swapped, and one/both substituted into equation / expression from (i) • 2 years = 24 months *High Partial Credit* • Fully substituted expression for c with logs/ equation in c with logs • Mixes up A and B, **or** mishandles years, and finishes correctly