Surds

$\sqrt{3} \simeq 1.73205808..$

It is irrational, this means that it can't be written in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$

the simplest way of writing $\sqrt{3}$ is $\sqrt{3}$

many numbers are most accurately written with a "$\sqrt{\ }$" we call these surds

SURDS CAN BE "SIMPLIFIED" IF THE NUMBER THE $\sqrt{\ }$ HAS A PERFECT SQUARE FACTOR

PERFECT SQUARES INCLUDE:

$\{4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 \ldots\}$

eg Simplify: $\sqrt{48}$? $\sqrt{48} = \sqrt{16(3)} = 4\sqrt{3}$

$\sqrt{75}$? $\sqrt{75} = \sqrt{25(3)} = 5\sqrt{3}$

$\sqrt{32}$? $\sqrt{32} = \sqrt{16(2)} = 4\sqrt{2}$

WHEN YOU SQUARE A SURD?

$$(\sqrt{2})^2 = 2$$

$$(\sqrt{5})(\sqrt{5}) = 5$$

$$(\sqrt{x})^2 = x$$

TO MULTIPLY BY AN INTEGER

$$(\sqrt{5})(6) = 6\sqrt{5}$$

SOMETIMES NUMBERS ARE WRITTEN WITH 2 OR MORE PARTS – THESE NUMBERS ARE CALLED COMPOUND NUMBERS

eg. $2+\sqrt{3}$

OR. $\sqrt{5}+2\sqrt{6}$

THE CONJUGATE OF A COMPLEX NUMBER.

NUMBER	$\longrightarrow$	CONJUGATE
$2+\sqrt{3}$	$\longrightarrow$	$2-\sqrt{3}$
$4-\sqrt{5}$	$\longrightarrow$	$4+\sqrt{5}$

When you multiply a compound number by its conjugate you often get a simple answer

Remember $(a+b)(a-b) = a^2 - b^2$ Difference of 2 squares

eg.. $(2-\sqrt{3})(2+\sqrt{3}) = 4 - 3 = 1$

This trick is used when dividing by a Compound Surd number

eg. $$\frac{3}{2+\sqrt{3}} = \frac{3(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

multiply above and below by the conjugate

$$= \frac{6-3\sqrt{3}}{4-3} = \frac{6-3\sqrt{3}}{1} = 6-3\sqrt{3}$$

Homework Solutions

1. Simplify each of the following:

(i) $\sqrt{8}$ $= \sqrt{4(2)}$ $= 2\sqrt{2}$

(ii) $\sqrt{27}$ $= \sqrt{9(3)}$ $= 3\sqrt{3}$

(iii) $\sqrt{45}$ $= \sqrt{9(5)}$ $= 3\sqrt{5}$

(iv) $\sqrt{200}$ $= \sqrt{100(2)}$ $= 10\sqrt{2}$

(v) $3\sqrt{18}$ $= 3\sqrt{9(2)}$ $= 9\sqrt{2}$

2. Express each of the following in its simplest form:

(i) $2\sqrt{2} + 6\sqrt{2} - 3\sqrt{2}$

$= 5\sqrt{2}$

(ii) $2\sqrt{2} + \sqrt{18}$

$= 2\sqrt{2} + \sqrt{9(2)}$

$= 2\sqrt{2} + 3\sqrt{2}$

$= 5\sqrt{2}$

(iii) $\sqrt{32} + \sqrt{18}$

$= \sqrt{16(2)} + \sqrt{9(2)}$

$= 4\sqrt{2} + 3\sqrt{2}$

$= 7\sqrt{2}$

(iv) $\sqrt{27} + \sqrt{48} - 2\sqrt{3}$

$= \sqrt{9(3)} + \sqrt{16(3)} - 2\sqrt{3}$

$= 3\sqrt{3} + 4\sqrt{3} - 2\sqrt{3}$

$= 5\sqrt{3}$

(v) $\sqrt{8} + \sqrt{200} - \sqrt{18}$

$= \sqrt{4(2)} + \sqrt{100(2)} - \sqrt{9(2)}$

$= 2\sqrt{2} + 10\sqrt{2} - 3\sqrt{2}$

$= 9\sqrt{2}$

(vi) $7\sqrt{5} + 2\sqrt{20} - \sqrt{80}$

$= 7\sqrt{5} + 2\sqrt{4(5)} - \sqrt{16(5)}$

$= 7\sqrt{5} + 4\sqrt{5} - 4\sqrt{5}$

$= 7\sqrt{5}$

TRICK- MULTIPLY ABOVE AND BELOW BY THE SURD DENOMINATOR

3. In each of the following quotients, rationalise the denominator.

(i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{2}{\sqrt{8}}$ (iii) $\frac{2}{5\sqrt{2}}$ (iv) $\frac{20}{\sqrt{50}}$ (v) $\frac{8}{\sqrt{128}}$

(i) $\frac{1(\sqrt{3})}{\sqrt{3}(\sqrt{3})} = \frac{\sqrt{3}}{3}$

(ii) $\sqrt{8} = \sqrt{4(2)} = 2\sqrt{2}$

$\frac{2}{\sqrt{8}} = \frac{2(\sqrt{8})}{(\sqrt{8})(\sqrt{8})} = \frac{2\sqrt{8}}{8} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$

(iii) $\frac{2}{5\sqrt{2}} = \frac{2(\sqrt{2})}{5\sqrt{2}(\sqrt{2})} = \frac{2\sqrt{2}}{10} = \frac{\sqrt{2}}{5}$

$\sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$

(iv) $\frac{20}{\sqrt{50}} = \frac{20\sqrt{50}}{\sqrt{50}\sqrt{50}} = \frac{20\sqrt{50}}{50} = \frac{2\sqrt{50}}{5} = \frac{2(5\sqrt{2})}{5} = 2\sqrt{2}$

$\sqrt{128} = \sqrt{64(2)} = 8\sqrt{2}$

(v) $\frac{8}{\sqrt{128}} = \frac{8}{8\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$

4. Simplify each of the following:

(i) $\sqrt{8} \times \sqrt{12}$ (ii) $3\sqrt{2} \times 5\sqrt{2}$ (iii) $\sqrt{2}(\sqrt{6} + 3\sqrt{2})$

(i)
$= \sqrt{8(12)}$
$= \sqrt{96}$
$= \sqrt{16(6)}$
$= 4\sqrt{6}$

(ii)
$= 15(2)$
$= 30$

(iii)
$= \sqrt{6(2)} + 3(2)$
$= \sqrt{12} + 6$
$= \sqrt{4(3)} + 6$
$= 2\sqrt{3} + 6$

(iv) $(5-\sqrt{3})(5+\sqrt{3})$ (v) $(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$ (vi) $(a+2\sqrt{b})(a-2\sqrt{b})$

$= 25-3$

$= 22$

$= 7-5$

$= 2$

$= a-4b$

Difference of 2 squares

5. By rationalising the denominator, express each of the following in its simplest form.

(i) $\dfrac{4}{\sqrt{5}+1}$ (ii) $\dfrac{12}{3-\sqrt{2}}$ (iii) $\dfrac{2-\sqrt{5}}{2+\sqrt{5}}$ (iv) $\dfrac{1}{\sqrt{8}-\sqrt{2}}$

(i) $\dfrac{4(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$= \dfrac{4\sqrt{5}-4}{5-1}$

$= \dfrac{4\sqrt{5}-4}{4}$

$= \sqrt{5}-1$

(ii) $\dfrac{12(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \dfrac{36+12\sqrt{2}}{9-2}$

$= \dfrac{36+12\sqrt{2}}{7}$

5. By rationalising the denominator, express each of the following in its simplest form.

(i) $\frac{4}{\sqrt{5}+1}$ (ii) $\frac{12}{3-\sqrt{2}}$ (iii) $\frac{2-\sqrt{5}}{2+\sqrt{5}}$ (iv) $\frac{1}{\sqrt{8}-\sqrt{2}}$

(iii) $\frac{(2-\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{4-4(5)+5}{4-5} = \frac{11}{-1} = -11$

(iv) $\frac{1 \quad (\sqrt{8}+\sqrt{2})}{(\sqrt{8}-\sqrt{2})(\sqrt{8}+\sqrt{2})} = \frac{\sqrt{8}+\sqrt{2}}{8-2} = \frac{\sqrt{4(2)}+\sqrt{2}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$