

Recap

| $f(x)$ | $f'(x)$ | Rules: |
|-------------------------------------|----------------------------|-----------------------------------|
| ax^n | anx^{n-1} | PRODUCT QUOTIENT CHAIN RULE |
| $\sin x$ | $\cos x$ | |
| $\cos x$ | $-\sin x$ | |
| $\tan x$ | $\sec^2 x$ | |
| $\sin^{-1}\left(\frac{x}{a}\right)$ | $\frac{1}{\sqrt{a^2-x^2}}$ | |
| $\tan^{-1}\left(\frac{x}{a}\right)$ | $\frac{a}{a^2+x^2}$ | |
| e^x | e^x | |
| e^{ax} | ae^{ax} | |
| $\ln x$ | $\frac{1}{x}$ | |

EXPONENTIAL & LOGARITHMIC FUNCTIONS DIFFERENTIATION

eg. 1

$$y = 5e^{2x}$$

$$\frac{dy}{dx} = 10e^{2x}$$

$$f(x) = e^{ax}$$

$$f'(x) = ae^{ax}$$

eg. 2

$$y = 2xe^x$$

PRODUCT $u = 2x$

$$\frac{du}{dx} = 2$$

$$v = e^x$$

$$\frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = 2xe^x + 2e^x$$

$$\text{eg. 3} \quad y = e^{(3x^2+2x)}$$

$$\frac{dy}{dx} = (e^{3x^2+2x})(6x+2)$$

$$f(x) = e^{ax}$$

$$f'(x) = ae^{ax}$$

$$\text{eg. 4} \quad y = e^{\cos x}$$

$$\frac{dy}{dx} = (e^{\cos x})(-\sin x)$$

$$\text{eg. 5} \quad y = 5e^{x^2}$$

$$\frac{dy}{dx} = (5e^{x^2})(2x) = 10xe^{x^2}$$

$$f(x) = e^{ax}$$

$$f'(x) = ae^{ax}$$

$$\text{eg. 6} \quad y = (e^x + 1)^4$$

$$\frac{dy}{dx} = 4(e^x + 1)^3 (e^x)$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

note: $\ln x = \log_e x$

eg.1 $y = \ln(4x^2 + 1)$

$$\frac{dy}{dx} = \frac{1(8x)}{(4x^2+1)} = \frac{8x}{4x^2+1}$$

eg.2

$$y = \log_e(\sin^2 x)$$

$$\frac{dy}{dx} = \frac{1(2\sin x)(\cos x)}{\sin^2 x}$$

$$= \frac{2\cos x}{\sin x} = 2\cot x$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

eq-3

$$y = \log_e 5x$$

$$y = \frac{1}{5x} = \frac{1}{x}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Homework - 28-9-2012

 $\frac{dy}{dx} = ?$

$$\textcircled{1} y = \ln(2x+3)$$

$$\textcircled{2} y = \frac{\ln x}{x}$$

$$\textcircled{3} y = \ln x (x^2+4)^2$$

$$\textcircled{4} y = \ln(\sqrt{\sin x})$$

$$\textcircled{5} y = e^{-3x}$$

$$\textcircled{6} y = e^{\sin^2 x}$$

$$\textcircled{7} y = x^2 e^x$$

$$\textcircled{8} y = -2e^x \sin x$$

Homework - 28-9-2012

 $\frac{dy}{dx} = ?$

① $y = \ln(2x+3)$

CHAIN RULE

$$\frac{dy}{dx} = \left(\frac{1}{2x+3}\right)(2) = \frac{2}{2x+3}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Homework - 28-9-2012

 $\frac{dy}{dx} = ?$

② $y = \frac{\ln x}{x}$

QUOTIENT RULE

$u = \ln x$

$v = x$

$\frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - 1(\ln x)}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Homework -28-9-2012

 $\frac{dy}{dx} = ?$

$$\textcircled{3} \quad y = \ln x (x^2 + 4)^2$$

PRODUCT RULE & CHAIN RULE

$$u = \ln x$$

$$v = (x^2 + 4)^2$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\frac{dv}{dx} = 2(x^2 + 4)(2x) = 4x(x^2 + 4)$$

$$\frac{dy}{dx} = 4x(x^2 + 4)\ln x + \frac{(x^2 + 4)^2}{x}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Homework -28-9-2012

 $\frac{dy}{dx} = ?$

$$\textcircled{4} \quad y = \ln(\sqrt{\sin x})$$

$$\text{note: } \sqrt{\sin x} = (\sin x)^{\frac{1}{2}}$$

CHAIN RULE

$$\frac{dy}{dx} = \left(\frac{1}{\sqrt{\sin x}} \right) \left(\frac{1}{2} (\sin x)^{-\frac{1}{2}} \right) (1)$$

$$= \frac{1}{2\sqrt{\sin x} \sqrt{\sin x}} = \frac{1}{2\sin x}$$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

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 $\frac{dy}{dx} = ?$

$$\textcircled{5} \quad y = e^{-3x}$$

$$\frac{dy}{dx} = -3e^{-3x}$$

$$f(x) = e^{ax}$$

$$f'(x) = a e^{ax}$$

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 $\frac{dy}{dx} = ?$

$$\textcircled{6} \quad y = e^{\sin^2 x}$$

CHAIN RULE

$$\frac{dy}{dx} = (e^{\sin^2 x}) (2 \sin x) (\cos x)$$

$$= 2 \sin x \cos x e^{\sin^2 x}$$

$$y = e^x$$

$$\frac{dy}{dx} = e^x$$

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 $\frac{dy}{dx} = ?$

$$\textcircled{7} \quad y = x^2 e^x \quad \text{PRODUCT RULE}$$

$$u = x^2 \quad v = e^x$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = x^2 e^x + 2x e^x$$

$$y = e^x$$

$$\frac{dy}{dx} = e^x$$

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 $\frac{dy}{dx} = ?$

$$\textcircled{8} \quad y = -2e^x \sin x \quad \text{PRODUCT RULE}$$

$$u = -2e^x \quad v = \sin x$$

$$\frac{du}{dx} = -2e^x \quad \frac{dv}{dx} = \cos x$$

$$\frac{dy}{dx} = -2e^x \cos x - 2e^x \sin x$$

$$y = e^x$$

$$\frac{dy}{dx} = e^x$$