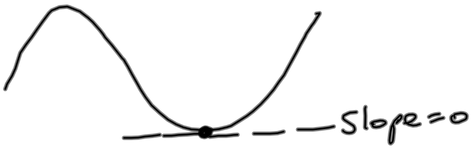
Applications of Differentiation



the slope of a tangent to a curve is $= 0$ at a turning point.

$f'(x) = \frac{dy}{dx} = 0$ at a turning pt.

To check if a turning point at $x = x_1$ is a maximum or a minimum

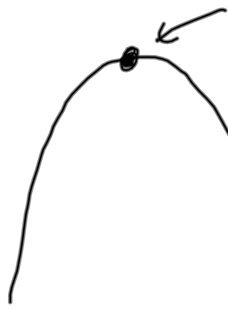
get the 2nd Derivative and sub. in x_1

if $f''(x_1) > 0$ [ie. positive] $\Rightarrow$ it is a minimum

if $f''(x_1) < 0$ [ie. negative] $\Rightarrow$ it is a maximum

Where is the turning point?

$$y = 2x^2 + 3x - 1 \qquad ?$$



$?\ \left(-\frac{3}{4}, -2\frac{1}{8}\right)$

$$\frac{dy}{dx} = 4x + 3$$

at turning pt. $\frac{dy}{dx} = 0$

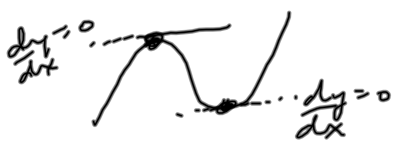
$$4x + 3 = 0$$
$$4x = -3$$
$$x = \frac{-3}{4}$$

$$y = 2\left(\frac{-3}{4}\right)^2 + 3\left(\frac{-3}{4}\right) - 1$$
$$= 2\left(\frac{9}{16}\right) - \frac{9}{4} - 1 = \frac{18}{16} - \frac{52}{16}$$
$$= \frac{-34}{16}$$

pt $\left(\frac{-3}{4}, \frac{-34}{16}\right)$

Find the turning points of

$$y = x^3 - 9x^2 + 15x$$



$$\frac{dy}{dx} = 3x^2 - 18x + 15 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-1)(x-5) = 0$$

$$x = 1, 5$$

$$y = (1)^3 - 9(1)^2 + 15(1) = 1 - 9 + 15 = 7 \qquad pt(1, 7) \text{ local max}$$

$$y = (5)^3 - 9(5)^2 + 15(5) = -25 \qquad pt(5, -25) \text{ local min}$$

TO PROVE IF THEY ARE MAX/MIN?

$$\frac{d^2y}{dx^2} = 6x - 18$$

$$\left.\frac{d^2y}{dx^2}\right|_{(x=1)} = 6(1) - 18 = -12 < 0 \quad \text{max}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(x=5)} = 6(5) - 18 = 12 > 0 \quad \text{min}$$

Homework Questions

(1) $y = x^2 - 4x + 9$ Find turning pt. and prove its a local min.

(2) $y = x^3 - 3x^2 - 9x + 6$ find 2 turning pts show which is max/min?

(3) Find the turning points of $y = \frac{x^2}{x+2}$

(4) Show $f(x) = \frac{x}{x+3}$ has no turning pts.

Find turning pt. Say if it is max/min?

(1) $y = x^2 - 4x + 9$

At turning pt

$\frac{dy}{dx} = 2x - 4 \Rightarrow 2x - 4 = 0$

$x = \frac{4}{2} = 2$

$y = (2)^2 - 4(2) + 9$

$= 4 - 8 + 9 = 4$

pt. $(2, 5)$

$\frac{d^2y}{dx^2} = 2 > 0 \Rightarrow$ curve has min.

(2) $y = x^3 - 3x^2 - 9x + 6$

Find turning pts. Say if they are max/min?

$\frac{dy}{dx} = 3x^2 - 6x - 9$

at turning pt $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1, 3$

$\Rightarrow y = (-1)^3 - 3(-1)^2 - 9(-1) + 6 = 11 \Rightarrow$ pt. $(-1, 11)$

$\Rightarrow y = (3)^3 - 3(3)^2 - 9(3) + 6 = -21 \Rightarrow$ pt $(3, -21)$

check max/min

$\frac{d^2y}{dx^2} = 6x - 6$

at $x=3 \Rightarrow 6(3) - 6 = 12 > 0 \Rightarrow$ min

at $x=-1 \Rightarrow 6(-1) - 6 = -12 < 0 \Rightarrow$ max

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use QUOTIENT RULE

$$y = \frac{x^2}{x+2}$$

$u = x^2$ $\quad v = x + 2$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{2x(x+2) - x^2}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$= \frac{x^2+4x}{(x+2)^2}$$

at turning pt $\Rightarrow$ $\frac{x^2+4x}{(x+2)^2} = 0$

multiply both sides by $(x+2)^2$

$\Rightarrow x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \mid x = -4$

$y = \frac{x^2}{x+2} \Rightarrow$

$x=0$ $\quad y = \frac{(0)^2}{0+2} = 0 \Rightarrow$ pt. $(0,0)$

$x=4$ $\quad y = \frac{(-4)^2}{(-4)+2} = \frac{16}{-2} = -8 \Rightarrow$ pt $(-4,-8$

(4) Show $f(x) = \frac{x}{x+3}$ has no turning pts.

if it had a turning point
then $f'(x) = 0$ at that point.

$f'(x) = ?$ use Quotient Rule

$u = x \qquad v = x+3$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 1$

$$f'(x) = \frac{(x+3)(1) - (x)(1)}{(x+3)^2} = \frac{x+3-x}{(x+3)^2}$$

$$= \frac{3}{(x+3)^2}$$

But this $\neq 0$
Since both the numerator and denominator are > 0

[note function does not exist when $x = -3$]

$\Rightarrow$ Since $f'(x) \neq 0$ it has no turning point.