

Algebra 1

Core Revision Questions

P.41

Q1

(i)

$$\frac{12m^2n^3}{(6m^4n^5)^2} = \frac{\cancel{12}m^2n^3}{\cancel{36}m^8n^{10}} = \frac{1}{3m^6n^7}$$

(ii)

$$\frac{3 + \frac{1}{x}}{\frac{5}{x} + 4} = \frac{3x + 1}{5 + 4x}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{2 + \frac{x}{2}}{x^2 - 16} = \frac{\cancel{2+x}}{2(x+4)(x-4)} \\
 & = \frac{1}{2x-8}
 \end{aligned}$$

Q2

$$\text{(i)} \quad y = x + 4$$

$$5y + 2x = 6$$

using substitution method (for change!)

$$5(x+4) + 2x = 6$$

$$5x + 20 + 2x = 6$$

$$7x = -14$$

$$x = -2$$

$$y = -2 + 4 = 2$$

Q2
(ii)

$$3x + y = 7 \Rightarrow y = 7 - 3x$$

$$x^2 + y^2 = 13$$

$$x^2 + (7 - 3x)^2 = 13$$

$$x^2 + 49 - 42x + 9x^2 = 13$$

$$10x^2 - 42x + 36 = 0$$

$$5x^2 - 21x + 18 = 0$$

$$(5x - 6)(x - 3) = 0$$

$$x = \frac{6}{5}, 3$$

$$y = 7 - 3\left(\frac{6}{5}\right)$$

$$= \frac{35 - 18}{5}$$

$$= \frac{17}{5} = 3\frac{2}{5}$$

$$\text{pt } (1\frac{1}{5}, 3\frac{2}{5})$$

$$y = 7 - 3(3) = -2$$

$$\text{pt } (3, -2)$$

Q3

$$\begin{array}{r}
 x^2 + 2x - 1 \\
 \hline
 x-3 | x^3 - x^2 - 7x + 3 \\
 \cancel{-x^3} \quad \cancel{+3x^2} \\
 \hline
 \cancel{+2x^2} - 7x \\
 \cancel{-7x^2} \quad \cancel{+6x} \\
 \hline
 \cancel{-x^2} + 3 \\
 \cancel{+x} \quad \cancel{+3} \\
 \hline
 0
 \end{array}$$

Q.4/

$$\begin{array}{r}
 3x^3 + 6x^2 + 3x + 33 \\
 \hline
 x - 2 \overline{) 3x^4 + 0x^3 - 9x^2 + 27x - 66} \\
 \underline{-3x^4 + 6x^3} \\
 \hline
 6x^2 - 9x^2 \\
 \underline{+ 6x^3 - 12x^2} \\
 \hline
 3x^2 + 27x \\
 \underline{- 3x^2 + 6x} \\
 \hline
 33x - 66 \\
 \underline{+ 33x + 66} \\
 \hline
 0
 \end{array}$$

Q.5/

Solve

$$x^4 - 9x^2 = 0$$

$$x^2(x^2 - 9) = 0$$

$$(x^2)(x-3)(x+3) = 0$$

$$\left| \begin{array}{c} x^2 = 0 \\ x = 0 \end{array} \right| \left| \begin{array}{c} x-3=0 \\ x=3 \end{array} \right| \left| \begin{array}{c} x+3=0 \\ x=-3 \end{array} \right|$$

Q5
(ii) Solve

$$(2x-1)^3(2-x) = 0$$

$$(a+b)^3$$

$$(a-b)^3$$

$$(2x-1)(4x^2+2x+1)(2-x) = 0$$

$2x-1 = 0$ $x = \frac{1}{2}$ \checkmark	$a=4$ $b=2$ $c=1$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-2 \pm \sqrt{4 - 16}}{8}$ PROBLEM NO REAL SOLUTION	$2-x = 0$ $2 = x$ \checkmark
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Q6

$$4x^2 + 20x + k$$

$$(2x + \overbrace{10x}^{\text{from } 20x})(2x + \overbrace{5}^{\text{from } k})$$

is a perfect square
k=?

$$4x^2 + 20x + \underline{25}$$

$$k=25$$

Q7

(i)

$$(3 - \sqrt{2})^2 = a - b\sqrt{2}$$

$$9 - 6\sqrt{2} + 2 = a - b\sqrt{2}$$

$$11 - 6\sqrt{2} = a - b\sqrt{2}$$

$$a = 11, \quad b = 6$$

Q7 *NOT READY FOR THIS YET!

(ii)

$$\frac{(1 - \sqrt{2})}{(1 + \sqrt{2})} = a\sqrt{2} - b$$

TRICK

MULTIPLY ABOVE AND BELOW
BY THE CONJUGATE OF
BOTTOM!

The conjugate of $1 + \sqrt{2} \Rightarrow 1 - \sqrt{2}$

$$\begin{aligned} \frac{(1 - \sqrt{2})(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} &= \frac{1 - 2\sqrt{2} + 2}{1 - 2} = \frac{3 - 2\sqrt{2}}{-1} \\ &= -3 + 2\sqrt{2} \end{aligned}$$

↑
DIFF
OF 2
SQUARES

Since
 $-3 + 2\sqrt{2} = a\sqrt{2} - b$
 $\Rightarrow a = 2, b = 3$

8. Factorise $x^3 - 27$

Difference of 2 cubes

$$(x-3)(x^2 + 3x + 9)$$

9. If $p(x - q)^2 + r = 2x^2 - 12x + 5$ for all values of x , find the values of p , q and r .

$$p(x^2 - 2qx + q^2) + r = \text{LHS}$$

$$px^2 - 2pqx + pq^2 + r = 2x^2 - 12x + 5$$

$$\Rightarrow p = 2$$

$$\begin{aligned} -2pq &= -12 \\ qp &= 6 \\ q(2) &= 6 \\ q &= 3 \end{aligned}$$

$$\begin{aligned} pq^2 + r &= 5 \\ (2)(3)^2 + r &= 5 \\ 18 + r &= 5 \\ r &= -13 \end{aligned}$$

- 10.** Solve the simultaneous equations
 ① $3x + 5y - z = -3$
 ② $2x + y - 3z = -9$
 ③ $x + 3y + 2z = 7$.

$$\begin{array}{r} \textcircled{3} \text{ by } -3 = \\ \underline{3x + 5y - z = -3} \\ -3x - 9y - 6z = -21 \\ \hline -4y - 7z = -24 \\ \textcircled{4} \quad 4y + 7z = 24 \end{array}$$

$$\begin{array}{r} \textcircled{2} \text{ by } -2 = \\ \underline{2x + y - 3z = -9} \\ -2x - 6y - 4z = -14 \\ \hline -5y - 7z = -23 \end{array}$$

$$\begin{array}{r} 5y + 7z = 24 \\ \underline{-4y - 7z = -23} \\ y = 1 \end{array}$$

$$\begin{array}{r} 5(1) + 7z = 24 \\ 7z = 19 \\ z = \frac{19}{7} = 2\frac{5}{7} \end{array}$$

$$\begin{array}{r} x + 3(1) + 2\left(\frac{19}{7}\right) = 7 \\ x = -\frac{10}{7} \end{array}$$

- 11.** Simplify $(b + 1)^3 - (b - 1)^3$.

Difference of 2 Cubes

$$\begin{aligned} &= ((b+1) - (b-1)) \left((b+1)^2 + (b+1)(b-1) + (b-1)^2 \right) \\ &= (2) \left(b^2 + 2b + 1 + b^2 - 1 + b^2 - 2b + 1 \right) \\ &= 2(3b^2 + 1) \\ &= 6b^2 + 1 \end{aligned}$$

- 12.** Find the rule for each of the following quadratic patterns.

- (i) 3, 12, 27, 48, 75 ...
 (ii) 5, 20, 45, 80, 125 ...
 (iii) 0.5, 2, 4.5, 8, 12.5 ...

PATTERN	$\textcircled{3} = b$	12	27	48	75
1st DIFF.		9	15	21	27
2nd DIFF.			$\textcircled{6} = 2a$	6	6

$$\Rightarrow a=3$$

$$\text{Rule : } ax^2 + b$$

$$\Rightarrow 3x^2 + 3$$

$$\text{Check: } f(3) = 3(3)^2 + 3 = 30 \times$$

Rule doesn't work!

$$\begin{array}{r}
 & 3 & 12 & 27 & 48 & 75 \\
 -3x^2 & -0 & -3 & -12 & -27 & -48 \\
 \hline
 & \textcircled{3} & 9 & 15 & & \\
 & \textcircled{6} & 6 & 6 & 6 & \\
 \hline
 \end{array}$$

LINEAR $ax+b$
 $= 6x+3$

ADD $3x^2$ to LINEAR \Rightarrow Rule is $3x^2 + 6x + 3$

$$\text{check: } f(3) = 3(3)^2 + 6(3) + 3 = 48 \checkmark$$

12. Find the rule for each of the following quadratic patterns.

- (i) 3, 12, 27, 48, 75 ...
- (ii) 5, 20, 45, 80, 125 ...
- (iii) 0.5, 2, 4.5, 8, 12.5 ...

PATTERN	5	20	45	80	125
1st DIFF.		15	25	35	45
2nd DIFF.			10	10	10

$a = 5$

$$\text{Rule: } ax^2 + b = 5x^2 + 5$$

Check: $f(0) = 0 \checkmark$
 $f(3) = 5(3)^2 + 5 = 50 \times$ Rule doesn't work.

$$\begin{array}{r}
 & 5 & 20 & 45 & 80 & 125 \\
 -5x^2 & -0 & -5 & -20 & -45 & -80 \\
 \hline
 & 5 & 15 & 25 & 35 & 45 & \text{LINEAR } ax+b \\
 & 10 & 10 & 10 & 10 & \hline
 & & & & & = 10x + 5
 \end{array}$$

LINEAR + $5x^2 \Rightarrow$ Rule is $5x^2 + 10x + 5$

Check: $f(4) = 5(4)^2 + 10(4) + 5 = 125 \checkmark$

12. Find the rule for each of the following quadratic patterns.

- (i) 3, 12, 27, 48, 75 ...
- (ii) 5, 20, 45, 80, 125 ...
- (iii) 0.5, 2, 4.5, 8, 12.5 ...

Pattern	0.5	2	4.5	8	12.5	QUADRATIC
1st Diff.		1.5	2.5	3.5	4.5	
2nd Diff			1	1	1	$a = \frac{1}{2}$

$$\text{Rule: } ax^2 + b = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\begin{aligned} \text{Check: } f(0) &= \frac{1}{2}(0)^2 + \frac{1}{2} = \frac{1}{2} \checkmark \\ f(4) &= \frac{1}{2}(4)^2 + \frac{1}{2} = \frac{1}{2}(16) + \frac{1}{2} = 8.5 \times \end{aligned}$$

So Rule doesn't work

Q12 Continued : We know its quadratic
 (iii)

	0.5	2	4.5	8	12.5	
$-\frac{1}{2}x^2$ term	-0	-0.5	-2	-4.5	-8	
	(0.5)	1.5	2.5	3.5	4.5	LINEAR
	(1)	1	1	1	1	$ax+b$ $= 1x + \frac{1}{2}$

ADD $\frac{1}{2}x^2$ to LINEAR \Rightarrow Rule is $\frac{1}{2}x^2 + x + \frac{1}{2}$

$$\text{check: } f(3) = \frac{1}{2}(3)^2 + 3 + \frac{1}{2} = 8 \checkmark$$

13. Find the rule for the pattern 6, 12, 20, 30, 42 using first and second differences. Hence find the 100th term of this pattern.

1st Difference	6	8	10	12		<u>QUADRATIC</u>
2nd Difference		(2)	2	2		
	$b = 6$		$2a = 2 \Rightarrow a = 1$			

$$\text{Rule: } ax^2 + b = x^2 + 6$$

$$\begin{aligned}
 f(0) &= 0^2 + 6 = 6 \quad \checkmark & \text{The sequence } 6, 7, 10, 15 \\
 f(1) &= 1^2 + 6 = 7 \quad \times & \text{is not the required} \\
 f(2) &= 2^2 + 6 = 10 \quad \times & \text{pattern!} \\
 f(3) &= 3^2 + 6 = 15 \quad \times
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 & 6 & 12 & 20 & 30 & 42 & & & \\
 \text{Subtract } x^2 \text{ term} & -6 & -1 & -4 & -9 & -16 & & & \\
 \hline
 & 11 & 16 & 21 & 26 & & & & \\
 & 5 & 5 & 5 & 5 & & & & \\
 \hline
 \end{array}
 \leftarrow \text{LINEAR} \qquad a = 5 \qquad b = 6$$

$$\text{LINEAR RULE: } 5x + 6$$

$$\text{ADD } x^2 \text{ TERM: } x^2$$

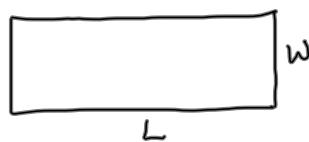
$$\text{QUADRATIC RULE: } x^2 + 5x + 6$$

$$\text{Check } f(0) = 6 \quad \checkmark$$

$$f(4) = (4)^2 + 5(4) + 6 = 42 \quad \checkmark$$

$$\Rightarrow \text{Term 100} = f(99) = (99)^2 + 5(99) + 6 = 10302$$

- 14.** Three times the width of a certain rectangle exceeds twice the length by 3 cm. Four times the length is 12 cm more than its perimeter. Find the dimensions of the rectangle.



$$3w - 2L = 3$$

$$\text{Perimeter} = 2L + 2w$$

$$4L - (2L + 2w) = 12$$

$$2L - 2w = 12$$

$$\begin{aligned} -2w + 2L &= 12 \\ 3w - 2L &= 3 \\ \hline w &= 15 \text{ cm} \end{aligned}$$

$$2L = 12 + 2w$$

$$2L = 12 + 2(15) = 42$$

$$L = 21 \text{ cm}$$

- 15.** The formula for a spherical mirror of radius r cm is given by $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$, where u cm is the object distance and v cm is the image distance to the mirror.

The magnification in the mirror is given by $m = \frac{v - r}{r - u}$.

- (i) Find r in terms of u and v .
(ii) Find m in terms of v and u only.

(i)

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

single fraction

$$\frac{u+v}{uv} = \frac{2}{r}$$

invert both sides

$$\frac{r}{2} = \frac{uv}{v+u}$$

multiply by 2^2

$$r = \frac{2uv}{v+u}$$

(ii)

$$m = v - \left(\frac{2uv}{v+u} \right)$$

$$\left(\frac{2uv}{v+u} \right) - u$$

Sub in r expression

$$m = \frac{v(v+u) - 2uv}{2uv - u(v+u)} = \frac{v^2 + uv - 2uv}{2uv - uv - u^2}$$

multiply by $(v+u)$

$$m = \frac{v^2 - uv}{uv - u^2} = \frac{v(1-u)}{u(v-1)}$$