Q4

$x^2 + 4x - 6 = (x+a)^2 + b$

$a = ?$
$b = ?$

$= x^2 + 2ax + a^2 + b$

$= x^2 + 2ax + (a^2 + b)$

$4 = 2a$
$a = 2$

$-6 = a^2 + b$
$-6 = 4 + b$
$b = -10$

Q5

$2x^2 + 5x + 6 = p(x+q)^2 + R$

$p = ?$
$q = ?$
$R = ?$

$= p(x^2 + 2qx + q^2) + R$

$= px^2 + 2pqx + pq^2 + R$

$p = 2$

$5 = 2pq$
$5 = 2(2)q$
$\frac{5}{4} = q$

$6 = pq^2 + R$
$6 = 2\left(\frac{5}{4}\right)^2 + R$
$6 = 2\left(\frac{25}{16}\right) + R$
$6 - \frac{50}{16} = R = \frac{23}{8}$

Q15

$(4x+R)(x^2+S) = 4x^3 + px^2 + qx + 2$, $pq = ?$

$4x^3 + 4Sx + Rx^2 + RS = RHS$

$4S = q$ $\quad R = p$ $\quad RS = 2$

$S = \frac{q}{4}$ $\quad p\left(\frac{q}{4}\right) = 2(4)$

$pq = 8$

Q11

$(x-4)^3 = x^3 + px^2 + qx - 64$ $\quad p = ?$ $\quad q = ?$

$(x-4)^2(x-4) = RHS$

$(x^2-8x+16)(x-4) = RHS$

$x^3 - 8x^2 + 16x - 4x^2 + 32x - 64 = RHS$

$x^3 - 12x^2 + 48x - 64 = RHS$

$p = -12$, $q = 48$

Example 3
p.20

$$\frac{1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$$

$A = ?$
$B = ?$

$$\text{LHS} = \frac{A(x-2) + B(x+1)}{(x+1)(x-2)}$$

$$A(x-2) + B(x+1) = 1$$

$$Ax - 2A + Bx + B = 1$$

$$(A+B)x + (B-2A) = 0x + 1$$

$$A + B = 0 \quad \Big| \quad B - 2A = 1$$

$$A = -B \quad \Big| \quad B - 2(-B) = 1$$

$$3B = 1$$

$$\Rightarrow B = \tfrac{1}{3}, \quad A = -\tfrac{1}{3}$$