Implicit Differentiation

Questions from Worksheet

Exercise 6E

① Find $\frac{dy}{dx}$:

(i) $x^2 + y^2 = 8$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y}$

(ii) $4x^2 - y^2 = 16$

$8x - 2y\frac{dy}{dx} = 0$

$8x = 2y\frac{dy}{dx}$

$\frac{4x}{y} = \frac{dy}{dx}$

(iii) $xy = 8$

$u = x \qquad v = y$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{dy}{dx}$

PRODUCT RULE:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\Rightarrow \quad x\frac{dy}{dx} + 1y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

(iv) $x^2 + xy + y^2 = 13$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -y - 2x$$

$$\frac{dy}{dx} = \frac{-y - 2x}{x + 2y}$$

1st Consider

Derivative of xy = ?

PRODUCT RULE:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x \qquad v = y$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{dy}{dx}$

Derivative $= x\frac{dy}{dx} + 1y$

(v) $5x - x^2y + 2 = 0$

$\Rightarrow 5 - x^2\frac{dy}{dx} - 2xy = 0$

$-x^2\frac{dy}{dx} = 2xy - 5$

$\frac{dy}{dx} = \frac{2xy-5}{-x^2}$

or $= \frac{5-2xy}{x^2}$

1st Consider

Derivative of x^2y = ?

PRODUCT RULE:

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2$ $\quad v = y$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = \frac{dy}{dx}$

Derivative = $x^2\frac{dy}{dx} + 2xy$

(vi) $2x^2 - 3xy + y^2 = 0$

$\Rightarrow 4x - 3\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$

$4x - 3x\frac{dy}{dx} - 3y + 2y\frac{dy}{dx} = 0$

$(2y-3x)\frac{dy}{dx} = 3y - 4x$

$\frac{dy}{dx} = \frac{3y-4x}{2y-3x}$

1st Consider

Derivative of xy = ?

PRODUCT RULE:

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x$ $\quad v = y$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = \frac{dy}{dx}$

Derivative = $x\frac{dy}{dx} + 1y$

Q2 $x^2+y^2=25$ Find $\frac{dy}{dx}$ at $(3,-4)$

$$2x+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\frac{dy}{dx}_{(3,-4)}=\frac{-3}{-4}=\frac{3}{4}$$

$x^2-xy+y^2=1$

Find $\frac{dy}{dx}$ at $(1,0)$.

$$\Rightarrow 2x-x\frac{dy}{dx}-y+2y\frac{dy}{dx}=0$$

$$(2y-x)\frac{dy}{dx}=y-2x$$

$$\frac{dy}{dx}=\frac{y-2x}{2y-x}$$

$$\frac{dy}{dx}_{(1,0)}=\frac{0-2(1)}{2(0)-1}=2$$

1st Consider

Derivative of xy = ?

PRODUCT RULE:

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$u=x$ $\quad v=y$

$\frac{du}{dx}=1$ $\quad \frac{dv}{dx}=\frac{dy}{dx}$

Derivative $=x\frac{dy}{dx}+1y$

Q4 Find slope of tangent to $x^2 - y^2 - x = 1$ at $(2,1)$

$$2x - 2y\frac{dy}{dx} - 1 = 0$$

$$-2y\frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} = \frac{1-2x}{-2y}$$

$$\frac{dy}{dx}(2,1) = \frac{1-2(2)}{-2(1)} = \frac{-3}{-2} = \frac{3}{2}$$

Q5 (i) Show $2x^2 - 3y^2 = 6$ contains $(-3,-2)$
(ii) Hence find equation of tangent at $(-3,-2)$

(i) Sub in $2(-3)^2 - 3(2)^2 \stackrel{?}{=} 6$

$2(9) - 3(4) \stackrel{?}{=} 6$

$18 - 12 \stackrel{?}{=} 6$

$6 = 6$

Yes so point is on curve

(ii) $4x - 6y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-4x}{-6y} = \frac{2x}{3y}$$

$$\frac{dy}{dx}(-3,-2) = \frac{2(-3)}{3(-2)} = \frac{-6}{-6} = 1 = \text{slope}$$

equation: $y - y_1 = m(x - x_1)$

$y - 2 = 1(x+3)$

$y - 2 = x + 3$

$x - y + 1 = 0$

Q6 Find slope of $x^2+3xy+2y^2=10$ at $(-1,3)$

$$2x+3\left(x\frac{dy}{dx}+y\right)+4y\frac{dy}{dx}=0$$

$$2x+3x\frac{dy}{dx}+3y+4y\frac{dy}{dx}=0$$

$$(3x+4y)\frac{dy}{dx}=-3y-2x$$

$$\frac{dy}{dx}=\frac{-3y-2x}{3x+4y}$$

$$\frac{dy}{dx}(-1,3)=\frac{-3(3)-2(-1)}{3(-1)+4(3)}$$

$$=\frac{-9+2}{-3+12}=\frac{-7}{9}$$

1st Consider

Derivative of xy = ?

PRODUCT RULE:

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$u=x$ $\quad v=y$

$\frac{du}{dx}=1$ $\quad \frac{dv}{dx}=\frac{dy}{dx}$

Derivative $=x\frac{dy}{dx}+1y$

Q8 Find slope of $(x-3)^2+(y+4)^2=5$ at $(1,-3)$

CHAIN RULE $\quad 2(x-3)(1)+2(y+4)\left(\frac{dy}{dx}\right)=0$ (DIVIDE BY 2)

$$(y+4)\frac{dy}{dx}=3-x$$

$$\frac{dy}{dx}=\frac{3-x}{y+4}$$

$$\frac{dy}{dx}(1,-3)=\frac{3-1}{-3+4}=2$$

Q9 $x^2 + y^2 - 4x - 6y + 9 = 0$ Find $\frac{dy}{dx}$.

$$2x + 2y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$(2y - 6)\frac{dy}{dx} = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4-2x}{2y-6} = \frac{2-x}{y-3}$$

Hence show $\frac{dy}{dx} = 0$ at $(2,1)$

$$\frac{dy}{dx}_{(2,1)} = \frac{2-2}{1-3} = 0 \quad \text{QED}$$

Q10 Find equation of tangent to $(x-2)^2 = 3y^2 + 2y$ at $(6,2)$

$$2(x-2)(1) = 6y\frac{dy}{dx} + 2\frac{dy}{dx}$$

$$2x - 4 = (6y + 2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x-4}{6y+2} = \frac{x-2}{3y+1}$$

$$\frac{dy}{dx}_{(6,2)} = \frac{6-2}{3(2)+1} = \frac{4}{7}$$

equation:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{4}{7}(x-6)$$

$$7y - 14 = 4x - 24$$

$$4x - 7y - 10 = 0$$