Differentiation by Rule

Rule 1

$$y = ax^n$$

$$\frac{dy}{dx} = anx^{n-1}$$

eg. 1

$$y = 3x^4$$

$$\frac{dy}{dx} = 3(4)x^{4-1}$$

$$= 12x^3$$

eg. 2

$$y = \sqrt{12x} = \sqrt{12}\ x^{\frac{1}{2}} = 2\sqrt{3}\ x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 2\sqrt{3}\left(\tfrac{1}{2}\right)x^{-\frac{1}{2}} = \sqrt{3}\ x^{-\frac{1}{2}} \checkmark$$

$$= \sqrt{3} \cdot \frac{1}{\sqrt{x}} = \frac{\sqrt{3}}{\sqrt{x}}$$

eg 3

$$y = 3x^2 + 2x - 1$$

$$\frac{dy}{dx} = 6x + 2 \checkmark$$

eg 4

$$y = 5x^3 + 2x^2 - 4x + 65$$

$$\frac{dy}{dx} = 15x^2 + 4x - 4 \checkmark$$

eg 5

$$y = 3x^5 + 2x$$

$$\frac{dy}{dx} = 15x^4 + 2$$

OR this can be written

$$f(x) = 3x^5 + 2x$$

$$f'(x) = 15x^4 + 2$$

eg 6

$$f(x) = y = 3x^3 + 4x^2 + 2x - 4$$

$$f'(x) = \frac{dy}{dx} = 9x^2 + 8x + 2$$

$$f''(x) = \frac{d^2y}{dx^2} = 18x + 8$$

(1) $y = x^3$ $\frac{dy}{dx} = 3x^2$	(4) $y = 3x$ $\frac{dy}{dx} = 3$	(7) $y = -3$ $\frac{dy}{dx} = 0$
(2) $y = 3x^4$ $\frac{dy}{dx} = 12x^3$	(5) $y = -2x$ $\frac{dy}{dx} = -2$	(8) $y = \frac{1}{x^2} = x^{-2}$ $\frac{dy}{dx} = -2x^{-3}$ ✓ or $\frac{-2}{x^3}$
(3) $y = -5x^2$ $\frac{dy}{dx} = -10x$	(6) $y = 5$ $\frac{dy}{dx} = 0$	(9) $y = \frac{2}{x^3} = 2x^{-3}$ $\frac{dy}{dx} = -6x^{-4}$

(10) $y = -\frac{2}{x^5} = -2x^{-5}$ $\frac{dy}{dx} = +10x^{-6}$	(12) $y = \frac{1}{x} = x^{-1}$ $\frac{dy}{dx} = -1x^{-2}$	(14) $y = \frac{4}{\sqrt{x}} = 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$
(11) $y = 6x^{\frac{1}{3}}$ $\frac{dy}{dx} = 2x^{-2/3}$	(13) $y = \sqrt{x} = x^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	(15) $y = \frac{1}{x^{2/3}} = x^{-2/3}$ $\frac{dy}{dx} = -\frac{2}{3}x^{-5/3}$

(16) $y = x^3 - 5x$

$\frac{dy}{dx} = 3x^2 - 5$

(17) $y = 1 - x^2$

$\frac{dy}{dx} = -2x$

(18) $y = x^2 - \frac{5}{x} = x^2 - 5x^{-1}$

$\frac{dy}{dx} = 2x + 5x^{-2}$

(19) $y = 2x^2 - \frac{3}{x^4} = 2x^2 - 3x^{-4}$

$\frac{dy}{dx} = 4x + 12x^{-5}$

(20) $y = \frac{1}{x^2} + \frac{1}{x} = x^{-2} + x^{-1}$

$\frac{dy}{dx} = -2x^{-3} - x^{-2}$

(21) $y = x^4 - \frac{2}{x^2} = x^4 - 2x^{-2}$

$\frac{dy}{dx} = 4x^3 + 4x^{-3}$

(22)

$y = 6\sqrt{x} - \frac{2}{\sqrt{x}}$

$= 6x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 3x^{-\frac{1}{2}} + x^{-3/2}$

(23)

$y = \frac{3}{x} + \frac{2}{x^2} + \frac{6}{x^{1/3}}$

$= 3x^{-1} + 2x^{-2} + 6x^{-1/3}$

$\frac{dy}{dx} = -3x^{-2} - 4x^{-3} - 2x^{-4/3}$

(24)

$y = \frac{2}{x} - \frac{1}{\sqrt{x}} + \frac{3}{x^{1/3}}$

$= 2x^{-1} - x^{-\frac{1}{2}} + 3x^{-1/3}$

$\frac{dy}{dx} = -2x^{-2} + \frac{1}{2}x^{-3/2} - x^{-4/3}$

Find $\frac{d^2y}{dx^2}$

(25)

$y = 4x^3 + 6x^2$

$\frac{dy}{dx} = 12x^2 + 6x$

$\frac{d^2y}{dx^2} = 24x + 6$

(26)

$y = x^2 - x^4$

$\frac{dy}{dx} = 2x - 4x^3$

$\frac{d^2y}{dx^2} = 2 - 12x$

(27)

$y = 6x^3 - 12x^2 - 8x + 4$

$\frac{dy}{dx} = 18x^2 - 24x - 8$

$\frac{d^2y}{dx^2} = 36x - 24$

Find $\frac{d^2y}{dx^2}$

(28)

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

$\frac{d^2y}{dx^2} = 2x^{-3}$

(29)

$y = x^2 - \frac{8}{x} = x^2 - 8x^{-1}$

$\frac{dy}{dx} = 2x + 8x^{-2}$

$\frac{d^2y}{dx^2} = 2 - 16x$

(30)

$y = \sqrt{x} = x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-3/2}$

Find $\frac{d^2y}{dx^2}$

(31)

$y = \frac{1}{\sqrt{x}} + \sqrt{x} = x^{-1/2} + x^{1/2}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2} + \frac{1}{2}x^{-1/2}$

$\frac{d^2y}{dx^2} = \frac{3}{4}x^{-5/2} - \frac{1}{4}x^{-3/2}$

(32)

$y = 8\sqrt{x} - \frac{1}{x^2} = 8x^{1/2} - x^{-2}$

$\frac{dy}{dx} = 4x^{-1/2} + 2x^{-3}$

$\frac{d^2y}{dx^2} = 2x^{-3/2} - 6x^{-4}$

(33)

$y = 9x^{1/3} + \frac{18}{x^{1/3}} = 9x^{1/3} + 18x^{-1/3}$

$\frac{dy}{dx} = 3x^{-2/3} - 6x^{-4/3}$

$\frac{d^2y}{dx^2} = -2x^{-5/3} + 8x^{-7/3}$

(34)

$f(x) = 3x^2 - 4x - 7$

(i) $f'(2) = ?$

$f'(x) = 6x - 4$

$f'(2) = 6(2) - 4 = 8$

(ii) $f''(-1) = ?$

$f''(x) = 6$

$f''(-1) = 6$

(35)

$f(x) = -4\sqrt{x} = -4x^{1/2}$

$f''(9) = ?$

$f'(x) = -2x^{-1/2}$

$f''(x) = x^{-3/2}$

$f''(9) = (9)^{-3/2}$

$= \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$

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$A = 3r^2 - 5r$

Find $\frac{dA}{dr}$ when $r=3$?

$\frac{dA}{dr} = 6r - 5$

$\frac{dA}{dr}_{(r=3)} = 6(3) - 5 = 18 - 5 = 13$

37

$s = 3t - 2t^2$

Find (i) $\frac{ds}{dt} = 3 - 6t$

$\frac{ds}{dt}_{(t=2)} = 3 - 6(2) = 3 - 12 = -9$

(ii) $\frac{d^2s}{dt} = -6$

$\frac{d^2s}{dt}_{(t=2)} = -6$

(38) $V = 3h - h^2 - 3h^3$

find $\frac{dV}{dh}$ when $h = 1$

$$\frac{dV}{dh} = 3 - 2h - 9h^2$$

$$\frac{dV}{dh}_{(h=1)} = 3 - 2(1) - 9(1)^2 = 3 - 2 - 9 = -8$$

(39) $A = \pi r^2$

Find $\frac{dA}{dr}$ when $\frac{r}{5} = 1$

$$\frac{dA}{dr} = 2\pi r$$

if $\frac{r}{5} = 1 \Rightarrow r = 5$

$$\frac{dA}{dr}_{(r=5)} = 2\pi(5) = 10\pi$$

(40)

$$V = \frac{4}{3}\pi r^3$$

Find $\frac{dV}{dr}$ when $2r - 5 = 0$?

$$\frac{dV}{dr} = 4\pi r^2$$

when $2r - 5 = 0 \Rightarrow 2r = 5 \Rightarrow r = 5/2$

$$\frac{dV}{dr}_{(r=5/2)} = 4\pi\left(\frac{5}{2}\right)^2 = 4\pi\left(\frac{25}{4}\right) = 25\pi$$

(41)

$$f(x) = 3x^2 - 4x$$

$f'(k) = 8$ FIND k?

$$f'(x) = 6x - 4$$

$$f'(k) = 6k - 4$$

Since $f'(k) = 8$

$$\Rightarrow 6k - 4 = 8$$

$$6k = 12$$

$$k = 2$$

(42) $f(x) = x^3 + 1$

$f''(a) = 18$ Find a?

$f'(x) = 3x^2$

$f''(x) = 6x$

$f''(a) = 6a$

Since $f''(a) = 18$

$\Rightarrow 6a = 18$

$a = 3$

(43) $y = 3x^2 + 2x$ Show $y\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 6x = 0$

$\frac{dy}{dx} = 6x + 2$

$\frac{d^2y}{dx^2} = 6$

$$y\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 6x = (3x^2 + 2x)(6) - 3x(6x+2) - 6x$$

$$= 18x^2 + 12x - 18x - 6x - 6x$$

$$= 0 \quad \text{QED}$$

(44) $y = 4x^3 - 6x^2$ show $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 12x^2 = 0$

$$\frac{dy}{dx} = 12x^2 - 6x$$

$$\frac{d^2y}{dx^2} = 24x - 6$$

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 12x^2 = x^2(24x-6) - 2x(12x^2-6x) - 12x^2$$

$$= 24x^3 - 6x^2 - 24x^3 + 18x - 12x^2$$

$$= 0$$

QED

(46) $y = \frac{1}{x^2}$ show $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 10y^3 = 0$

$y = x^{-2}$ $\frac{dy}{dx} = -2x^{-3}$ $\frac{d^2y}{dx^2} = +6x^{-4}$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 10y^3 = x^{-2}(6x^{-4}) + (-2x^{-3})^2 - 10(x^{-2})^3$$

$$= 6x^{-6} + 4x^{-6} - 10x^{-6}$$

$$= 10x^{-6} - 10x^{-6}$$

$$= 0$$

QED