Algebra

Special Factors
$(x^2 - y^2) = (x - y)(x + y)$
$(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$
$(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$

Special Expansions
$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$
$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

Surds
Properties of Surds:

1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
3. $\sqrt{a}\sqrt{a} = a$

Rationalise Denominator
$$\frac{\mathbf{-10 + 6\sqrt{3}}}{\mathbf{1 + \sqrt{3}}}$$
$$= \frac{-10 + 6\sqrt{5}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

Forming a Quadratic Equation
$x^2 - (sum\ of\ the\ roots)x + (product\ of\ the\ roots) = 0$

Sum and Product of the Roots of a Quadratic
The quadratic equation $ax^2 + bx + c = 0$ can be written $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

If α and β are the roots of $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, then:

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Useful $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ identity
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Laws of Indices
Properties of Indices

1. $a^m . a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m+n}$
4. $(ab)^n = a^n b^n$
5. $a^{-n} = \frac{1}{a^n}$
6. $a^{\frac{1}{n}} = \sqrt[n]{a}$
7. $a^0 = 1$

Dividing Fractions
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Solving Quadratics
$f(x) = 2x^2 - 4x - 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots
$(k - 1)x^2 - 6x + (k - 1)$ has equal roots, find k.

Real	$b^2 - 4ac \geq 0$
Equal	$b^2 - 4ac = 0$
No Real Roots	$b^2 - 4ac < 0$

Laws of Logs
$\log_a m + \log_a n = \log_a mn$
$\log_a m - \log_a n = \log_a \frac{m}{n}$
$n\log_a m = \log_a m^n$
$\log_n m = \frac{\log_a m}{\log_a n}$

Log Equations
$\log_2(5x + 1) = 2\log_2(x + 1)$

Apply above rules to solve

The Unknown in the Power
(Easy) $2^x = 8$

Use Logs ⟶ $\log_2 8 = x$

(Hard) $2^x - 6 + 2^{3-x} = 0$

Let $y = 2^x$ *and try make quadratic.*

Solving Cubics
Solve $f(x) = 4x^3 + 10x^2 - 7x - 3$

We must guess 1st root (if not given) by subbing in for x*. It will be factor of 3*
Then if $x = -3$ *is a root* $x + 3$ *is a factor and can be used in long division to solve.*

Using Factor Theorem
$x - 1$ and $x - 2$ are factors of $f(x) = ax^3 + bx^2 + x + 2$. Find a and b

If $x - 1$ *is a factor then* $x = 1$ *is a root and we can sub this into equation.*

Identities/ Unknown Co-efficients
$(x+a)^2-(x+b)^2=6x+24$

Remove all fractions or brackets. Equate like terms on each side.

Irrational Equations
$x-\sqrt{2x-4}=2$

Isolate the surd. Square both sides and solve. May have to repeat. Always test solution.

Modulus/ Absolute Value
$|x-2|\geq 3$

Isolate the modulus and then square both sides

If $|x|=4$

then $x=4$
or $x=-4$

Rational Inequalities
$$\frac{3x+1}{x+1}\leq 1$$

Cannot cross multiply as $x+1$ may be negative. Multiply each side by $(x+1)^2$. Solve the quadratic to get critical values.
If $\leq$ then it's between values. If $\geq$ then it's outside values.

Abstract Inequalities

Show that $a^2+b^2\geq 2ab$

Any real number squared is non-negative.
Bring to one side and factorise to make something squared.

Fractions
Show that $\frac{5}{x-2}+\frac{26-7x}{2-x}$ reduces to a constant

Find a common denominator.

Simultaneous Equations – 2 unknowns (linear)
Solve $2x+8y=10$
$2x-3y=-1$

Multiply one or both lines to make co-efficients of one of the variables the same. Cancel down and solve.

Occurs in co-ordinate geometry to find where lines intersect

Simultaneous Equations – 3 unknowns
Solve $2x+8y-3z=-1$
$2x-3y+2z=2$
$2x+y+z=5$

We take equations in pairs and eliminate one variable.

Simultaneous Equations – 2 unknowns (linear & non-linear)
Solve $2x+y=10$
$x^2+y^2-4x-2y=0$

Take the linear expression and express one variable in terms of the other. Sub this into the non-linear and solve.

Occurs in co-ordinate geometry where a line intersects a circle

Substitution

$$\sqrt{\frac{q^2+rp+r+4}{\frac{-q}{p}}}$$

For $p=3, q=-4$ and $r=7$

Sub in values for p, q and r

Manipulate Formulae

Express a in terms of u, t and s

$$s=ut+\frac{1}{2}at^2$$

Get rid of brackets and fractions.
Isolate letter of choice.