chapter 7

Algebra 3

Section 7.6 Indices

x^n

Power
exponent
index

Base

PROJECT MATHS
Text & Tests 6

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Example 1

Evaluate each of the following.

(i) $27^{\frac{1}{3}}$ (ii) $36^{\frac{3}{2}}$ (iii) $64^{-\frac{2}{3}}$ (iv) $\left(\frac{27}{125}\right)^{-\frac{2}{3}}$

$27^{1/3} = \sqrt[3]{27} = 3$

$36^{3/2} = \left(\sqrt{36}\right)^3 = 6^3 = 216$

$64^{-2/3} = \dfrac{1}{\left(\sqrt[3]{64}\right)^2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

$\left(\dfrac{27}{125}\right)^{-2/3} = \left(\dfrac{125}{27}\right)^{2/3} = \left(\sqrt[3]{\dfrac{125}{27}}\right)^2 = \left(\dfrac{5}{3}\right)^2 = \dfrac{25}{9}$

Log Tables p.21

$x^{-1} = \dfrac{1}{x}$
"Reciprocal"

Indices:
Adding Rule
Subtracting Rule
Multiplication Rule

- $(2^3)(2^2) = 2^5$ Adding Rule
- $\frac{2^5}{2^3} = 2^2$ Subtracting Rule
- $\frac{2^6}{2^8} = 2^{-2}$ Subtracting Rule
- $(2^5)(2^{-3}) = 2^2$ Adding Rule
- $(2^3)^3 = 2^9$ Multiplication Rule

Example 2

Simplify each of the following. (i) $\left(\frac{x^2y^{-3}}{x^{-4}y^5}\right)^{\frac{1}{2}}$ (ii) $\frac{\sqrt{a^3}}{\sqrt[4]{a} \times \sqrt[3]{a^2}}$

(i) $\left(\frac{x^2y^{-3}}{x^{-4}y^5}\right)^{\frac{1}{2}}$

$= \left(x^{2--4}\, y^{-3-5}\right)^{\frac{1}{2}}$

$= (x^6y^{-8})^{\frac{1}{2}}$

$= x^3y^{-4}$

(ii) $\frac{\sqrt{a^3}}{\sqrt[4]{a}\,.\sqrt[3]{a^2}}$

$= \frac{a^{3/2}}{a^{1/4}.a^{2/3}}$

$= \frac{a^{3/2}}{a^{1/4+2/3}} = \frac{a^{3/2}}{a^{11/12}}$

$= a^{3/2-11/12} = a^{7/12}$

Example 3

Show that $\frac{5^{n+1} - (4)5^n}{5^{n-2} + 5^n} = \frac{25}{26}$.

$5^{n+1} = 5(5^n)$

FACTORISE

$$\Rightarrow \frac{5(\cancel{5^n}) - 4(\cancel{5^n})}{\frac{\cancel{5^n}\,1}{5^2} + \cancel{5^n}\,1} \stackrel{?}{=} \frac{25}{26}$$

$$\frac{1}{\frac{1}{5^2} + 1} \stackrel{?}{=} \frac{25}{26}$$

$$\frac{1}{5^2} + 1 \stackrel{?}{=} \frac{26}{25}$$

$$\frac{1}{25} + \frac{1}{1} = \frac{26}{25} \quad \checkmark \text{ true}$$

5. Express $\frac{4^2 \times 16^{\frac{1}{2}}}{64^{\frac{2}{3}} \times 4^3}$ in the form $4^n, n \in Z$.

$2^4 = 16$

$2^n = 16, \quad n = ?$

NO.

$\log_{\boxed{2}} \boxed{16} = 4$

BASE

$$n = \log_{\boxed{4}} \boxed{\frac{4^2 \times 16^{1/2}}{64^{2/3} \times 4^3}} = -2$$