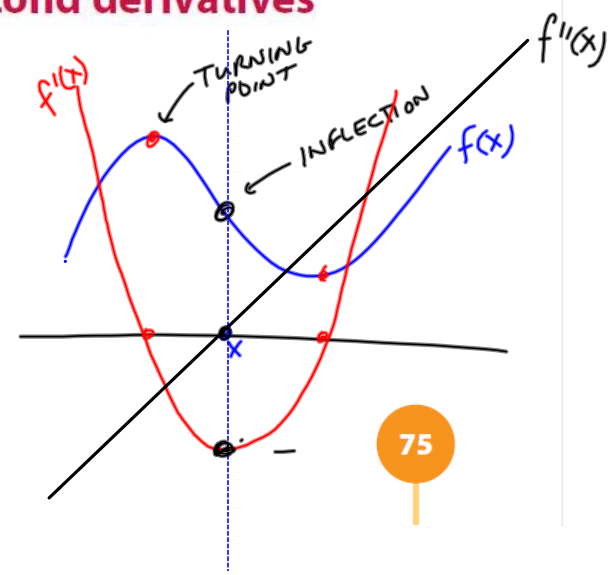
Differential Calculus

chapter 2

Section 2.5 Second derivatives

$f''(x)$

$\frac{d^2y}{dx^2}$



PROJECT MATHS Text & Tests 7

75

For any function $y = f(x)$, the first derivative is $\frac{dy}{dx}$ or $f'(x)$.

If we differentiate the resulting function, we get the **second derivative**.

The second derivative is denoted by $\mathbf{\frac{d^2y}{dx^2}}$ or $\boldsymbol{f''(x)}$.

$\frac{d^2y}{dx^2}$ is pronounced 'dee two y dee x squared'.

We learned earlier that for any function $y = f(x)$, $\frac{dy}{dx}$ represents the slope of the tangent to the curve at any point on the curve. When dealing with the graphs of functions in the next chapter, we will see that $\frac{d^2y}{dx^2}$ gives the rate at which the slope is changing over a given interval.

Example 1

Given that $y = x + \frac{1}{x}$, find $\frac{d^2y}{dx^2}$.

$y = x + \frac{1}{x} = 1x + x^{-1}$

$\frac{dy}{dx} = 1 - 1x^{-2}$

$\frac{d^2y}{dx^2} = 2x^{-3}$

Example 2

If $y = \frac{3}{x} + 4x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$; hence, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

$y = \frac{3}{x^1} + 4x = 3x^{-1} + 4x$

1st derivative $\frac{dy}{dx} = -3x^{-2} + 4$

2nd derivative $\frac{d^2y}{dx^2} = 6x^{-3}$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ $\Rightarrow$ $x^2(6x^{-3}) + x(-3x^{-2}+4) - (3x^{-1}+4x) = 0$

$6x^{-1} - 3x^{-1} + 4x - 3x^{-1} - 4x = 0$

$0 = 0$ ✓