Differential Calculus

chapter 2

Section 2.7 Differentiation of inverse trigonometric functions

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Standard derivatives of inverse functions

$$f(x) = \sin^{-1}\left(\frac{x}{a}\right) \Rightarrow f'(x) = \frac{1}{\sqrt{a^2 - x^2}}$$

$$f(x) = \tan^{-1}\left(\frac{x}{a}\right) \Rightarrow f'(x) = \frac{a}{a^2 + x^2}$$

Example 1

If $y = \sin^{-1}\frac{5x}{3}$, find $\frac{dy}{dx}$.

$a = \frac{3}{5}$

$$y = \sin^{-1}\left(\frac{5x}{3}\right) = \sin^{-1}\left(\frac{x}{\left(\frac{3}{5}\right)}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}$$

$$= \frac{1}{\sqrt{\frac{9}{25} - x^2}}$$

Standard derivatives of inverse functions

$$f(x) = \sin^{-1}\left(\frac{x}{a}\right) \Rightarrow f'(x) = \frac{1}{\sqrt{a^2 - x^2}}$$

$$f(x) = \tan^{-1}\left(\frac{x}{a}\right) \Rightarrow f'(x) = \frac{a}{a^2 + x^2}$$

Example 2

If $y = \tan^{-1}(2x + 1)$, find $\frac{dy}{dx}$.

Chain Rule

outside : $\tan^{-1}\left(\frac{y}{a}\right)$

inside : $(2x+1)$

$f(x) = \tan^{-1}\frac{(2x+1)}{1}$ $\boxed{a=1}$

$$f'(x) = \frac{1}{(1)^2 + (2x+1)^2} \cdot (2)$$

$$= \frac{2}{1 + 4x^2 + 4x + 1} = \frac{2}{4x^2 + 4x + 2}$$

$$= \frac{1}{2x^2 + 2x + 1}$$