Differential Calculus

chapter 2

Section 2.8 Differentiation of exponential functions

$f(x) = e^x$
$f'(x) = e^x$

$f(x) = e^{ax}$
$f'(x) = ae^{ax}$

Basic Rule

$f(x) = e^{(\text{whatever})}$
$f'(x) = \left[e^{(\text{whatever})}\right](\text{diff of whatever})$

PROJECT MATHS Text & Tests 7

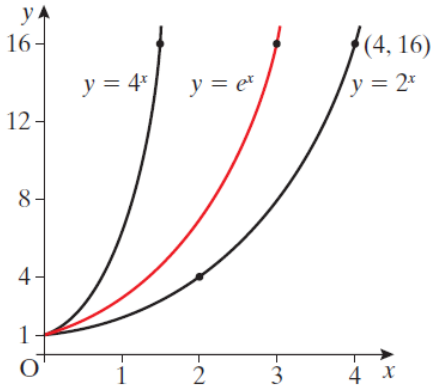
The word **exponent** is often used instead of *index*.
A function such as $y = 2^x$, where the variable occurs in the index, is called an **exponential function**.

The diagram on the right shows the graphs of these three functions
(i) $y = 2^x$ (ii) $y = e^x$ (iii) $y = 4^x$

The function illustrated by the red curve is $\boldsymbol{y = e^x}$.
It is by far the most important exponential function.

It is generally referred to as **the** exponential function.
e is an irrational number whose value is 2·718, correct to three decimal places.

The importance of the function $y = e^x$ lies in the fact that the slope at any point on the curve is always the same as the y-value of the function.



If $y = e^x$, then $\frac{dy}{dx} = e^x$.

Example 1

Find $\frac{dy}{dx}$ for each of the following:

(i) $y = 5e^{x^2}$ (ii) $y = e^{\cos x}$ (iii) $y = (e^x + 1)^4$

$y = 5e^{x^2}$

$\frac{dy}{dx} = 5(e^{x^2})(2x) = 10xe^{x^2}$

$y = e^{\cos x}$

$\frac{dy}{dx} = (e^{\cos x})(-\sin x) = -\sin x e^{\cos x}$

$y = (e^x + 1)^4$

$\frac{dy}{dx} = 4(e^x + 1)^3(e^x) = 4e^x(e^x + 1)^3$

Example 2

If $y = e^{2x}\cos 2x$, find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{8}$.

PRODUCT

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = e^{2x}$

$\frac{du}{dx} = 2e^{2x}$

$v = \cos 2x$

$\frac{dv}{dx} = (-\sin 2x)(2)$

$= -2\sin 2x$

$\frac{dy}{dx} = (e^{2x})(-2\sin 2x) + (\cos 2x)(2e^{2x})$

$= 2e^{2x}(\cos 2x - \sin 2x)$

$f(\frac{\pi}{8}) = 2e^{2(\frac{\pi}{8})}(\cos 2(\frac{\pi}{8}) - \sin 2(\frac{\pi}{8}))$

$= 0$