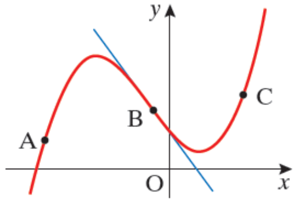
Points of Inflection

The curve traced in the diagram below is said to be **concave upwards** from the point A to the point B, and **concave downwards** from the point B to the point C.



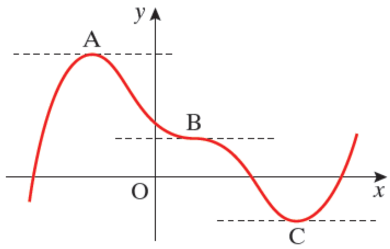
The point B, where the curve changes from being concave upwards to concave downwards, is called a **point of inflection**.

At a point of inflection, the tangent to the curve crosses the curve.

At a point of inflection, B, on the curve $y = f(x)$, $\frac{d^2y}{dx^2} = 0$ and changes sign as the curve passes through B.

Note: The point B on the given curve is a point of inflection. The tangent to the curve at B is also parallel to the x-axis.

The point B is called a **saddle point** or a **horizontal point of inflection.**



Thus, to find the point(s) of inflection of a curve:

(i) find $\frac{d^2y}{dx^2}$

(ii) solve the equation $\frac{d^2y}{dx^2} = 0$

(iii) for each value of x, find the corresponding value of y.

Example 3

Find the point of inflection of the curve $y = x^3 - 3x^2 - 2$.

$$\frac{dy}{dx} = 3x^2 - 6x$$

at inflection pt. $\frac{d^2y}{dx^2} = 0$

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\Rightarrow 6x - 6 = 0$$
$$x - 1 = 0$$
$$x = 1$$

$x=1,\ y=?$

$$y = (1)^3 - 3(1)^2 - 2 = 1 - 3 - 2 = -4$$

pt $(1, -4)$

8. Find the turning point of the function $y = x - \sqrt{x}\ (x \geqslant 0)$ and determine the nature of this turning point.

at turning pt. $\frac{dy}{dx} = 0$

at max $\frac{d^2y}{dx^2} < 0$

at min $\frac{d^2y}{dx^2} > 0$

$$y = x - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-\frac{1}{2}}$$

$$1 - \frac{1}{2}x^{-\frac{1}{2}} = 0$$

$$-\frac{1}{2\sqrt{x}} = -1$$

$$\frac{1}{2} = \sqrt{x}$$

$$x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

10. Show that the curve $y = \cos x$ has a point of inflection at $x = \frac{\pi}{2}$.

at inflection pt.

$\frac{d^2y}{dx^2} = 0$

$y = \cos x$

$\frac{dy}{dx} = -\sin x$

$\frac{d^2y}{dx^2} = -\cos x$

$\Rightarrow -\cos x = 0$

$\cos x = 0 \quad \Rightarrow x = \cos^{-1}(0) = \frac{\pi}{2}$

Is $\cos\frac{\pi}{2} = 0$?

$\cos\left(\frac{\pi}{2}\right) = 0$ ✓ $\Rightarrow$ inflection pt.