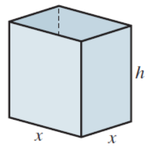
**5.** A storage tank in the shape of a cuboid has a capacity of $108\text{ m}^3$. It has a square base of side $x$ metres with vertical sides and open at the top.

(i) Express the height, $h$, in terms of $x$.

(ii) Show that the surface area, $S$, is given by $S = x^2 + \frac{432}{x}$.

(iii) Find the dimensions of the tank if the surface area is to be a minimum.




$V = LBH \Rightarrow$ $\quad 108 = x^2 h \Rightarrow h = \frac{108}{x^2}$

$S = 2[LB + BH + LH]$ $\quad S = 2\left[x^2 + \cancel{x}\left(\frac{108}{x^{\cancel{2}}}\right) + \cancel{x}\left(\frac{108}{x^{\cancel{2}}}\right)\right]$

$= 2\left[x^2 + \frac{216}{x}\right]$

$S = 2x^2 + \frac{432}{x} \quad \Rightarrow S = 2x^2 + 432x^{-1}$

$\frac{dS}{dx} = 0$ at min

$\Rightarrow \frac{dS}{dx} = 4x - 432x^{-2} = 0$

$\Rightarrow 4x^3 - 432 = 0$

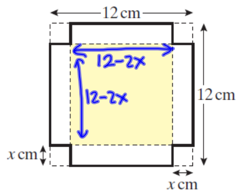
$x^3 = \frac{432}{4} = 108$

$x = \sqrt[3]{108} = 4.762$

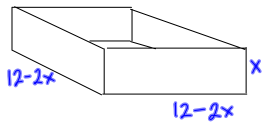
**6.** A square sheet of card of side 12 cm has four equal squares of side $x$ cm cut from the corners, as shown. The sides are then turned up to make an open rectangle box.

(i) Express the volume of the box in terms of $x$.

(ii) Show that the volume of the box is a maximum when $x$ is 2.




$V = LBH$




$V = (12-2x)^2(x)$

$= (144 - 48x + 4x^2)x$

$= 4x^3 - 48x^2 + 144x$

when $V$ is max

$\frac{dV}{dx} = 0$

$\frac{dV}{dx} = 12x^2 - 96x + 144$

$\Rightarrow 12x^2 - 96x + 144 = 0$

$\div 12$ $\quad x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

$\Rightarrow x = 6,\ x = 2$

at max

$\frac{d^2V}{dx^2} < 0$

$\frac{d^2V}{dx^2} = 24x - 96$

$x = 2$ $\quad \Rightarrow \frac{d^2V}{dx^2} = 24(2) - 96 = 48 - 96 = -48 < 0$

$\Rightarrow$ max at $x = 2$